NON-STATIONARY FLOW BETWEEN TWO PARALLEL PLATES WITH THE GAP PARTIALLY FILLED WITH A POROUS MEDIA

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SUMMARY

Stationary and non - stationary flows are of great interest from the practical view point. We present in this work an exact solution of the equations that describe the flow between two parallel plates with the gap partially filled with a porous media. The upper plate oscillates longitudinally while the lower one remains stationary. Analytical expressions for the velocities and temperatures are derived and analysed as functions of porosity and thermal conductivity.

1. Introduction

Engineering applications such as petroleum wells drilling, flows in porous media, elastic waves propagation in soils, etc., have motivated a great deal of interest on stationary and non-stationary flows between parallel plates. Several authors(Ishgaki, 1971 - Cox, 1991) have determined velocity and temperature profiles of these flows modeled by the Navier-Stokes equations. Despite the large number of works, in some specific cases of lubrification mechanics, such as journal bearings - where the surfaces are porous material and there is a lubrificating film between them - exact solutions are hardly found in the literature.

The present work is a generalization of Stokes' second problem studied by Schlichting (1968) and it complements the study of Carrocci (1982).

We present the analysis of a viscous, incompressible flow between two parallel, horizontal plates. The space between them is partially filled with an isotropic nonhomogeneous porous media. The upper one oscillates longitudinally while the lower one remains stationary (see Fig. 2.1).

In section 2 we model the problem and section 3 shows the numerical results obtained solving the corresponding equations.

In the fourth section we discuss the results.

2. Mathematical Model

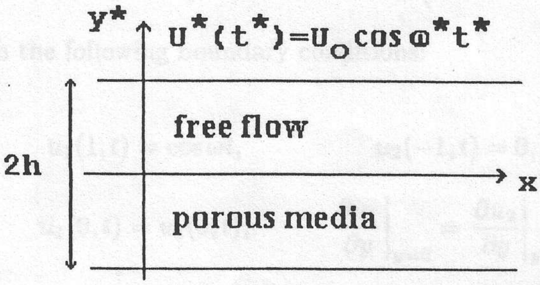

Figure 2.1: Geometrical description of the problem. $U^*(t^*)$ represents the velocity of the upper plate.

Consider a two-dimensional flow of an incompressible fluid between two parallel infinite plates separated by a distance $2h$. The upper plate oscillates longitudinally and the lower one remains stationary, as shown in Figure 1. Suppose that the flow does not change in the x direction and that there is no pressure gradient acting on the flow. The analysis of the problem must be divided accordingly to the two regions, one in which the fluid flows freely and the other for the flow through the porous media.

Let us denote with 1 and 2 the variables corresponding to the free flow and in the porous media respectively, then let u_j^* be the flow velocities, ν_j the kinematic viscosities and ε the permeability of the porous media . If the upper plate's motion is given by $U^*(t^*) = U_0 \cos \omega^* t^*$ with U_0 and ω^* constants, then the momentum equations are:

free flow

$$\frac{\partial u_1^*}{\partial t^*} = \frac{\partial U^*(t^*)}{\partial t^*} + \nu_1 \frac{\partial^2 u_1^*}{\partial y^{*2}}, \tag{2.1}$$

flow in the porous media

$$\frac{\partial u_2^*}{\partial t^*} = \frac{\partial U^*(t^*)}{\partial t^*} + \nu_2 \frac{\partial^2 u_2^*}{\partial y^{*2}} + \frac{\nu_2}{\varepsilon}\left[U^*(t^*) - u_2^*\right], \tag{2.2}$$

subjected to the following boundary conditions

$$u_1^*(h, t^*) = U^*(t^*), \qquad u_2^*(-h, t^*) = 0,$$

$$u_1^*(0, t^*) = u_2^*(0, t^*), \qquad \left.\frac{\partial u_1^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial u_2^*}{\partial y^*}\right|_{y^*=0}.$$

Under the same flow hypothesis defined before, considering the heat transfer by convection zero, and denoting by T_j^* the temperatures, κ_j and c_j the thermal conductivities and specific heat constants, the equations for the temperature distributions are:

$$\frac{\partial T_j^*}{\partial t^*} = \kappa_j \frac{\partial^2 T_j^*}{\partial y^{*2}} + \frac{\nu_j}{c_j}\left(\frac{\partial u_j^*}{\partial y^*}\right)^2, \tag{2.3}$$

with the boundary conditions

$$\begin{aligned} T_1^*(h, t^*) &= T_2^*(-h, t^*) = T_1, \\ T_1^*(0, t^*) &= T_2^*(0, t^*), \\ \kappa_1 \left.\frac{\partial T_1^*}{\partial y^*}\right|_{y^*=0} &= \kappa_2 \left.\frac{\partial T_2^*}{\partial y^*}\right|_{y^*=0}. \end{aligned}$$

Let us define the dimensionless parameters

$$\begin{aligned} &t = t^* \frac{\nu_1}{h^2}, \qquad y = \frac{y^*}{h}, \\ &\omega = \omega^* \frac{h^2}{\nu_1}, \qquad U = \frac{U^*}{U_0}, \\ &\alpha_j = \frac{\nu_j}{\nu_1}, \\ &\beta_1 = 0, \qquad \beta_2 = \frac{\nu_2 h^2}{\nu_1 \varepsilon}, \\ &u_j = \frac{u_j^*}{U_0}, \\ &\theta_j = \frac{T_j^* - T_0}{T_1 - T_0}, \end{aligned} \tag{2.4}$$

then, using (2.4) in (2.1),(2.2) and (2.3) we finally obtain the equations for the velocities and temperature:

$$\frac{\partial u_j}{\partial t} = \frac{\partial U(t)}{\partial t} + \alpha_j \frac{\partial^2 u_j}{\partial y^2} + \beta_j \left[U(t) - u_j\right], \tag{2.5}$$

$$\frac{\partial \theta_j}{\partial t} = \frac{\alpha_j}{\mathrm{Pr}_j} \frac{\partial^2 \theta_j}{\partial y^2} + \alpha_j Ec_j \left(\frac{\partial u_j}{\partial y}\right)^2, \tag{2.6}$$

valid for $j = 1, 2$. In (2.6), Pr_j and Ec_j represent the Prandtl and Eckert numbers for the regions.

It can be observed that although the system is nonlinear because equation (2.6) is coupled with (2.5) by the quadratic term, it can be studied with a linear analysis, solving first (2.5) and using the result as a non homogeneity in (2.6).

Equation (2.5) is solved utilizing the method of separation of variables, assuming a general solution of the form

$$\begin{aligned} u_j(y,t) = \cos\omega t + \cos\omega t\ &[c_{1j} \cos\sigma_j y \cosh\sigma_j y \\ &+ c_{2j} \sin\sigma_j y \cosh\sigma_j y \\ &+ c_{3j} \cos\sigma_j y \sinh\sigma_j y] \\ &+ c_{4j} \sin\sigma_j y \sinh\sigma_j y \\ + \sin\omega t\ &[c_{5j} \cos\sigma_j y \cosh\sigma_j y \\ &+ c_{6j} \sin\sigma_j y \cosh\sigma_j y \\ &+ c_{7j} \cos\sigma_j y \sinh\sigma_j y \\ &+ c_{8j} \sin\sigma_j y \sinh\sigma_j y], \end{aligned} \tag{2.7}$$

where

$$\sigma_1 = Re\left(\sqrt{\imath\omega}\right), \qquad \sigma_2 = Re\left(\sqrt{\frac{\beta + \imath\omega}{\alpha}}\right),$$

with the following boundary conditions:

$$u_1(1, t) = \cos\omega t, \qquad u_2(-1, t) = 0,$$

$$u_1(0, t) = u_2(0, t), \qquad \left.\frac{\partial u_1}{\partial y}\right|_{y=0} = \left.\frac{\partial u_2}{\partial y}\right|_{y=0}.$$

Solving (2.5),(2.6) is reduced to solve a linear algebraic system, for which we used a program written in the symbolic manipulation language Mathematica (Wolfram,1988). We developed the study of the described problem for the cases: a) the two plates have given, constant temperatures. b) the upper plate is adiabatic and the lower one is isothermic and c) the upper plate is isothermic and the lower one adiabatic using the following boundary conditions:

a)

$$\theta_1(1,t) = 1, \qquad \theta_2(-1,t) = 0,$$

$$\theta_1(0,t) = \theta_2(0,t), \qquad \kappa_1 \left.\frac{\partial \theta_1}{\partial y}\right|_{y=0} = \kappa_2 \left.\frac{\partial \theta_2}{\partial y}\right|_{y=0}.$$

b)

$$\left.\frac{\partial \theta_1}{\partial y}\right|_{y=1} = 0, \qquad \theta_2(-1,t) = 0,$$

$$\theta_1(0,t) = \theta_2(0,t), \qquad \kappa_1 \left.\frac{\partial \theta_1}{\partial y}\right|_{y=0} = \kappa_2 \left.\frac{\partial \theta_2}{\partial y}\right|_{y=0}.$$

c)

$$\theta_1(1,t) = 0, \qquad \left.\frac{\partial \theta_2}{\partial y}\right|_{y=-1} = 0,$$

$$\theta_1(0,t) = \theta_2(0,t), \qquad \kappa_1 \left.\frac{\partial \theta_1}{\partial y}\right|_{y=0} = \kappa_2 \left.\frac{\partial \theta_2}{\partial y}\right|_{y=0}.$$

3. Results

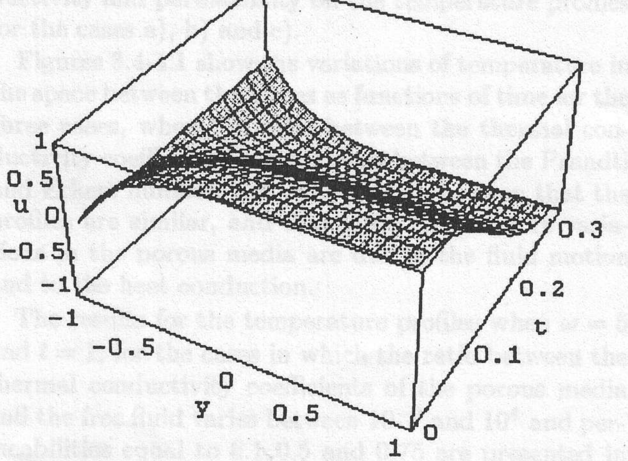

Figure 3.1: Velocity u as function of time and space for $\omega = 10$ and $\varepsilon = 0.1$.

We present in this section the results obtained evaluating numerically the solutions of equations (2.5) and (2.6) yield by the program written in Mathematica commented before.

Figure 3.1 shows the evolution in time of the velocity profiles of case a) with $\omega = 10$ and $\varepsilon = 0.1$. One can observe in it that, due to the oscillation of the upper plate, the plane velocity profiles corresponding to a uniform flow are deformed into a curved surface.

The analysis of the velocity's behavior on a time interval equal to a half-period of oscillation $T/2 = \pi/\omega$,

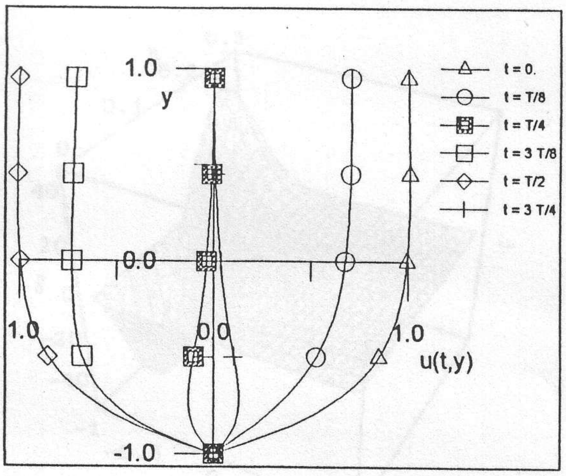

Figure 3.2: Dependence of u on time, with y varying between the plates in case a). The parameters are $\omega = 10$, $\varepsilon = 0.1$.

with $\omega = 10$ is summarized in Figure 3.2. It is shown there that even in the absence of an external pressure gradient, inversion of motion is verified at several instants, for example at $t = T/4$. These inversions are consequences of the viscous forces and the inertia of the fluid.

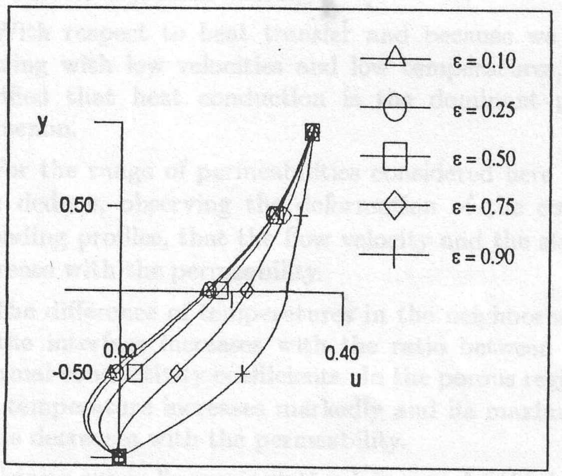

Figure 3.3: Dependence of the velocity u on the permeability ε in the case a) of two isothermic plates.

The dependence of the velocity on the permeability is presented in Figure 3.3, for which we used a frequency $\omega = 5$ and ε varying between 0.1 and 0.9. It can be observed that the inertia forces in the porous media are two orders of magnitude smaller than the viscous forces, verifying Darcy's law (Batchelor, 1967).

We next illustrate the influences of the thermal con-

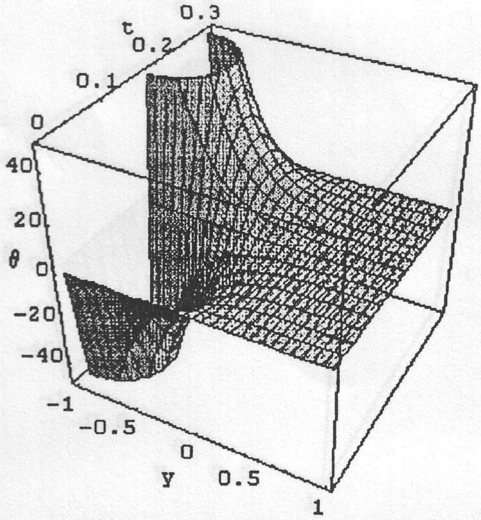

Figure 3.4: Dependence of θ on time in case c) for y varying between the plates. The frequency is $\omega = 10$, $\varepsilon = 0.1$ and $\kappa = 1$.

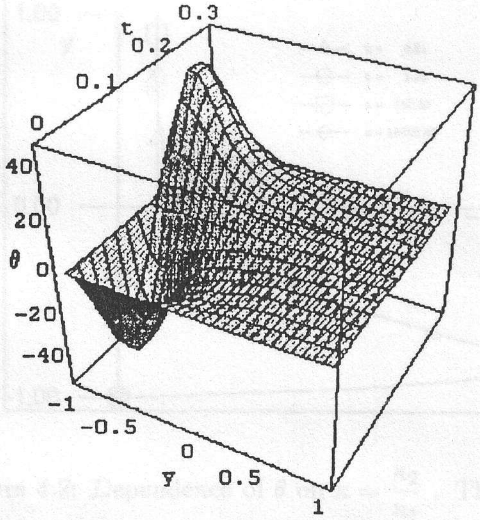

Figure 3.5: Temperture θ as a function of space and time in case a) of two isothermic plates.

ductivity and permeability on the temperature profiles for the cases a), b) and c).

Figures 3.4-4.1 show the variations of temperature in the space between the plates as functions of time for the three cases, when the ratio between the thermal conductivity coefficients and the ratio between the Prandtl and Eckert numbers are unity. It can be seen that the profiles are similar, and that large temperature variations in the porous media are due to the fluid motion and to the heat conduction.

The results for the temperature profiles when $\omega = 5$ and $t = 1$, for the cases in which the ratio between the thermal conductivity coefficients of the porous media and the free fluid varies between 10^{-2} and 10^{4} and permeabilities equal to 0.1,0.5 and 0.75 are presented in figures 4.2 and 4.3. It is verified that the maximum temperature of the flow in the porous media, at that instant, decreases with the permeability. It can also be observed that the difference between the thermal conductivity coefficients changes the temperature, producing the separation of the curves at the interface between the two media.

4. Comments and Conclusions

The values between 5 and 20 for the dimensionless frequency were established through numerical experiments, utilizing a program written in Mathematica. The range of permeabilities between 0.1 and 0.5 is of particular interest in Tribology because it better represents real engineering cases and, thus, we choose these values for the analysis in this paper.

We have determined that high permeabilities, besides diminishing the lubrificant fluid retention, also diminish the dumping property of the porous media to mechanical shocks in journal bearings.

With respect to heat transfer and because we are dealing with low velocities and low temperatures, we verified that heat conduction is the dominant phenomenon.

For the range of permeabilities considered here one can deduce, observing the deformation of the corresponding profiles, that the flow velocity and the stress increase with the permeability.

The difference of temperatures in the neighborhood of the interface increases with the ratio between the thermal conductivity coefficients. In the porous region, the temperature increases markedly and its maximum value decreases with the permeability.

Using a symbolic manipulation language for this kind of problem yields interesting results from the physical point of view. From the practical point of view, the results are also encouraging because one can obtain fast and very accurate values. For example, the determination of the numerical solutions for the velocity and temperature for this paper were obtained in $50s$ and $100s$ respectively.

Acknowledgments

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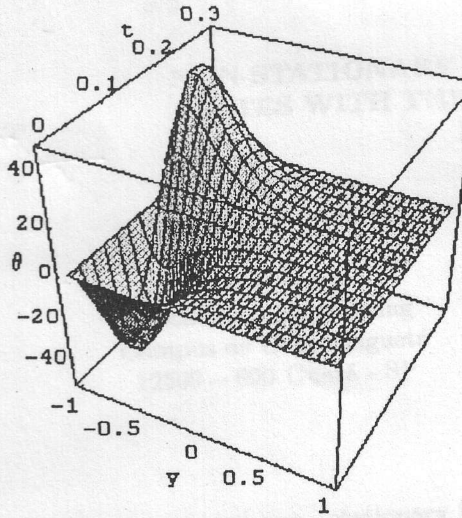

Figure 4.1: Temperature θ as a function of space and time in case b) with $\omega = 10$ and $\varepsilon = 0.1$.

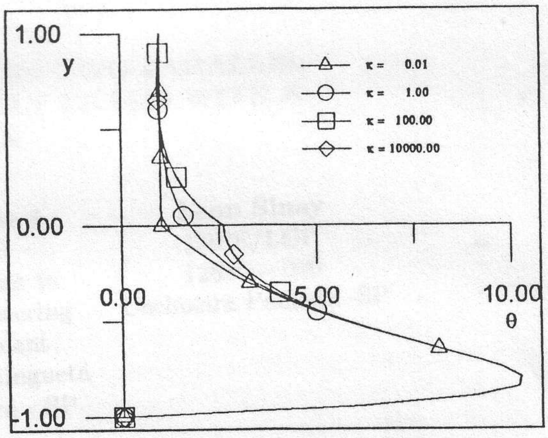

Figure 4.2: Dependence of θ on $\kappa = \dfrac{\kappa_2}{\kappa_1}$. The permeability is $\varepsilon = 0.1$.

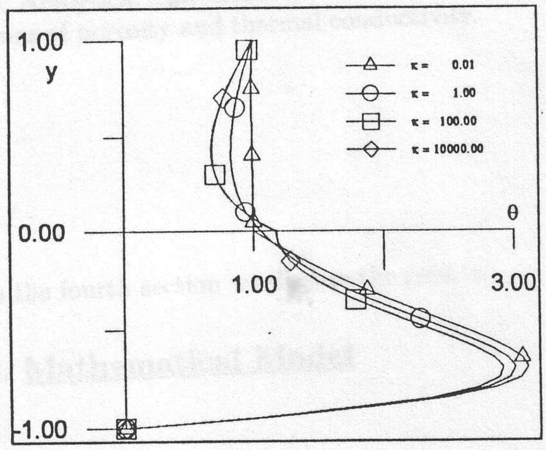

Figure 4.3: Idem as Figure 4.2 with $\varepsilon = 0.5$. It can be observed an almost singular behavior at $y = 0$.

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