0147

AIAA 95-3115

Cylindrical Diffusion Laminar Flames

D. Bastos-Netto
INPE
Cachoeira Paulista, SP, Brazil

31st AIAA/ASME/SAE/ASEE
Joint Propulsion Conference and Exhibit
July 10-12, 1995/San Diego, CA

CYLINDRICAL DIFFUSION LAMINAR FLAMES

D.Bastos-Netto*
Instituto Nacional de Pesquisas Espaciais, INPE
12630-000, Cachoeira Paulista, SP, BRAZIL

Abstract

Axisymmetric tubular laminar flames are investigated in a near equilibrium diffusion controlled regime with a thin reaction zone separating the fuel and oxidizer radial streams. The solution of this "normal Burke-Schumann like problem" yields the flame position and the concentration profiles for the fuel and oxidizer and, as expected, the proper stretching variable for future investigation of the flame structure for large activation energies. The temperature profile is also obtained for the classical conditions of Lewis Numbers equal to unity.

Introduction

The importance of flame-flow interaction in Combustion Research is undeniable. While premixed flames have been intensively studied in a wide spectrum of geometries and conditions[1-6] this is not so with diffusive flames. Following the work of Burke and Schumann[7] who set forth the theory of diffusional combustion and the work of Shvab and Zel'dovich[8], who established the general properties of diffusion flames, most of the work has been concentrated in the counter-flow diffusion flames[9-10] and in the spherical diffusion flames[11-12] the former used by Liñán[10] in laying forth the use of the activation-energy asymptotics technique for diffusion flames and the latter because of the fuel drops burning problems. Cylindrical diffusion laminar flames have not been studied yet, may be due to the fact stated by Buckmaster and Ludford[13] that "the uniformity of the far field implied by the cylindrical operator precludes the possibility of a flux of oxidant from infinity so that, as in the plane case, such a diffusion flame cannot exist". However, this might not be the case if one chooses a geometry and conditions as described in the next section which can be experimentally realized. Hence this work, which uses Burke and Schumann's idea of an infinitely thin chemical reaction zone, i.e., the combustion surface approximation as shown by Zeldovich et al[8], to establish the flame position, the temperature and the concentration profiles to lay the path for future investigation on the asymptotic structure of these flames.

* Senior Researcher, Combustion and Propulsion Laboratory

Member AIAA

Problem Geometry

Consider the sketch shown in Figure 1 where a stream of gaseous fuel (subscript 2) flows radially outwards through an infinitely long porous circular cylinder of radius r_2 while the gaseous oxidizer (subscript 1) is being pumped radially inwards through another porous cylinder of radius r_1, $(r_1 >> r_2)$, concentric with the inner cylinder. Assume a single step exothermic chemical reaction in which the oxidizer A_1 and the fuel A_2 are converted into the combustion product A_3,

$$\nu_1 A_1 + \nu_2 A_2 \rightarrow \nu_3 A_3 \tag{1}$$

where ν_i are the stoichiometric coefficients (i=1,2,3). The flow is steady, axisymmetric and symmetric about the plane z=0. Assuming the usual simplifying conditions[8,14] the Conservation Equations can be written:

mass:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{\partial}{\partial z}(\rho v_z)=0 \tag{2}$$

r-mom.:

$$\rho\left(v_r\frac{\partial v_r}{\partial r}-\frac{v_\theta^2}{r}+\frac{\partial v_r}{\partial z}v_z\right)+\frac{\partial p}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(2\frac{\partial v_r}{\partial r}-\frac{2}{3}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}\right)\right)\right.\ -$$

$$-\frac{2\mu v_r}{r^2}+\frac{2}{3}\frac{\mu}{r}\left(\frac{1}{r}\frac{\partial}{\partial r}rv_r+\frac{\partial v_z}{\partial z}\right)$$

$$\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right] \tag{3}$$

θ-mom.:

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\mu r^3\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v_\theta}{\partial z}\right) \tag{4}$$

z-mom:

$$\rho\left[v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right]+\frac{\partial p}{\partial z}$$

$$=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right)+$$

$$+\frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial v_z}{\partial z}-\frac{2}{3}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}\right)\right)\right. \tag{5}$$

energy:

$$\rho\left(v_r\frac{\partial h}{\partial r}+v_z\frac{\partial h}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\lambda}{c_p}r\frac{\partial h}{\partial r}\right)$$

$$+\frac{\partial}{\partial z}\left(\frac{\lambda}{c_p}\frac{\partial h}{\partial z}\right)+QW \tag{6}$$

species i, (i=1,2):

$$\rho\left(v_r\frac{\partial a_i}{\partial r}+v_z\frac{\partial a_i}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\rho D_i r\frac{\partial a_i}{\partial r}\right)$$

$$+\frac{\partial}{\partial z}\left(\rho D_i\frac{\partial a_i}{\partial z}\right)-\nu_i W \tag{7),(8}$$

combustion product, (i=3):

$$\rho\left(v_r\frac{\partial a_3}{\partial z}+v_z\frac{\partial a_3}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\rho D_3 r\frac{\partial a_3}{\partial r}\right)$$

$$+\frac{\partial}{\partial z}\left(\rho D_3\frac{\partial a_3}{\partial z}\right)+\nu_3 W \tag{9}$$

where v_r, v_θ and v_z are the velocity components and r, p, μ, h, λ, c_p and D are the gas density, pressure, dynamic viscosity coefficient, enthalpy, thermal conductivity, constant pressure specific heat and diffusivity and where Q is the chemical source, $a_i = \rho_i/\rho$ is the relative concentration of species A_i (i=1,2,3), and it has been assumed that the reaction rate for the ith component, W_i, can be written in terms of the chemical reaction rate, W, using the stoichiometric coefficients ν_i, so that:

$$W=-{W_i}/{\nu_i} \quad = {W_3}/{\nu_3} \quad (i=1,2) \tag{10}$$

where $W_i = da_i/dt$, (i=1,2). Also choose a_{i0}, (i=1,2) to be the initial oxidizer and fuel relative concentrations, respectively.

The Combustion Surface

Define

$$\bar{p} = \frac{a_1}{\nu_1} - \frac{a_2}{\nu_2} \tag{11}$$

and assume that $\rho D_i = \rho D$, i = 1,2 i.e., on either side of the flame sheet[8]. Then Equations (7) and (8) yield

$$\rho\left(v_r\frac{\partial \bar{p}}{\partial r} + v_z\frac{\partial \bar{p}}{\partial z}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\rho D r\frac{\partial \bar{p}}{\partial r}\right) + \frac{\partial}{\partial z}\left(\rho D\frac{\partial \bar{p}}{\partial z}\right) \tag{12}$$

where

$$\bar{p}(r_2, z) = -\frac{a_{20}}{\nu_2} \qquad \forall \quad z \tag{12.a}$$

$$\bar{p}(r_1, z) = \frac{a_{10}}{\nu_1} \qquad \forall \quad z \tag{12.a'}$$

and $\bar{p} = 0$ at the Combustion Surface, $r = r_F$ [8], i.e., $\bar{p}(r_F, z) = 0$. Since $r_1 >> r_2$, then condition (12.a') can be conveniently substituted by

$$\bar{p}(\infty, z) = \frac{a_{10}}{\nu_1} \qquad \forall \quad z \tag{12.b}$$

For the present geometry and away from the plane z = 0, then $\bar{p} = \bar{p}(r)$ only and equations (12), (12.a) and (12.b) can be written as:

$$\rho v_r\frac{d\bar{p}}{dr} = \frac{1}{r}\frac{d}{dr}(r\rho D\frac{d\bar{p}}{dr}) \tag{12.c}$$

and

$$\bar{p}(r_2) = -\alpha_{20} \tag{12.d}$$

$$\bar{p}(\infty) = \alpha_{10} \tag{12.e}$$

where $\alpha_{i0} = \dfrac{a_{i0}}{\nu_i}$ (i = 1,2).

The velocity field

Introducing independent variables η_i such that

$$\eta_i^2 = \frac{\bar{a}_i}{\nu_{i0}} r^2 \tag{13}$$

where the $\bar{a}_i$ are rate of strain parameters and ν_{i0} the kinematic viscosities (i = 1 for the oxidant flow (i.e., for $r > r_s$) and i = 2 for the fuel flow (i.e., for $r_2 < r < r_s$)). Then the radial velocity, v_r, can be written as

$$\rho\, v_r\, r = -\rho_{i0}\, \nu_{i0}\, f_i(\eta_i) \tag{14}$$

where $f_i(\eta_i)$ are streamfunctions. Then Eq. (5) is transformed in

$$\frac{f}{\eta}\left(\frac{f'}{\eta}\right)' + \left(\frac{f'}{\eta}\right)^2 - 1 = \frac{1}{\eta}\left[\eta\left(\frac{f'}{\eta}\right)'\right]' \tag{15}$$

$$\lim_{\eta \to \eta_{s\pm}} f(\eta) =$$

where the subscript i was dropped for convenience and whose solution is:

$$f(\eta) = \frac{\eta^2 - \eta_S^2}{2} \tag{16}$$

Therefore

$$v(r) = -\frac{\bar{a}_i}{2}\left(\frac{r^2 - r_S^2}{r}\right) \quad , \ i = 1,2 \tag{17}$$

and the axial velocity will be given by

$$v_z(z) = \bar{a}_i z\frac{f(\eta)'}{\eta} \quad \text{or} \quad v_z(z) = \bar{a}_i z \tag{18}$$

Equation (12.c) can be written as

$$-f(\eta)\frac{d\bar{p}}{d\eta} = \frac{1}{Sc_i}\frac{d}{d\eta}(\eta\frac{d\bar{p}}{d\eta}) \tag{19}$$

where $Sc_i = \mu_{0i}/\rho D_i$

Let $\xi=\eta/\eta_S$, (so that $\xi \to \xi_F$ as $\eta \to \eta_{F+or-}$ on either side of the flame). Then equations (19) yield

$$\xi\bar{p}'' + [(1-A_i) + A_i\xi^2]\bar{p}' = 0, \qquad (20)$$

$(') = d/d\xi$

where $A_i = \frac{Sc_i}{2}\eta_S^2 \qquad (21)$

and $\bar{p}$ must satisfy the conditions:

$$\begin{aligned}&\bar{p}(\xi_{F+}) = 0 \quad \textit{and}\\ &\lim_{\xi \to \infty} \bar{p} = \alpha_{10} \quad \textit{for} \quad \xi > \xi_{F+}\end{aligned} \qquad (22)$$

$$\begin{aligned}&\bar{p}(\xi_{F-}) = 0\\ &\textit{and} \quad \bar{p}(\xi_0) = \alpha_{20} \textit{for} \quad \xi_0 < \xi < \xi_{F-}\end{aligned} \qquad (23)$$

where

$$\xi_0 = \frac{\eta_0}{\eta_S}, \quad \eta_0^2 = \frac{\bar{a}_2}{\nu_{20}} r_2^2.$$

The solution of Eq. (20) subjected to conditions (22) is

$$\bar{p} = \alpha_{10} \frac{\left[\gamma\left(\frac{A_1}{2}, \frac{A_1}{2}\xi^2\right) - \gamma\left(\frac{A_1}{2}, \frac{A_1}{2}\xi_F^2\right)\right]}{\Gamma\left(\frac{A_1}{2}\right) - \gamma\left(\frac{A_1}{2}, \frac{A_1}{2}\xi_F^2\right)} \qquad (24)$$

$, \ \xi > \xi_F$

and the solution satisfying (23) is

$$\bar{p} = \alpha_{20} \frac{\left[\gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi^2\right) - \gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi_F^2\right)\right]}{\left[\gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi_F^2\right) - \gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi_0^2\right)\right]} \qquad (25)$$

$, \ \xi_0 < \xi < \xi_F$

where γ and Γ are the Incomplete Gamma Function and the Gamma Function respectively [15].

Imposing continuity of $\frac{d\bar{p}}{d\xi}$ *at* $\xi = \xi_F$, one obtains

$$\gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi^2\right) - \gamma\left(\frac{A_2}{2}, \frac{A_2}{2}\xi_0^2\right)$$

$$\frac{\alpha_{20}}{\alpha_{10}}\left(\frac{A_2}{2}\right)^{\frac{A_2}{2}}\left(\frac{A_1}{2}\right)^{-\frac{A_1}{2}} e^{-\left(\frac{A_2-A_1}{2}\right)\xi_F^2}. \qquad (26)$$

$$\left[\Gamma\left(\frac{A_1}{2}\right) - \gamma\left(\frac{A_1}{2}, \frac{A_1}{2}\xi_F^2\right)\right]\xi_F^{A_2-A_1}$$

that is, an equation for ξ_F as a function of ξ_0. It can be shown that, if the fuel and the oxidizer have the same properties so that $A_i = A$ and the variable ξ can be taken as continuous, then the condition for the existence of a single value of ξ_F is given by:

$$\gamma\left(\frac{A}{2}, \frac{A}{2}\xi_F^2\right) = \frac{\alpha_{10}}{\alpha_{10}+\alpha_{20}}\gamma\left(\frac{A}{2}, \frac{A}{2}\xi_0^2\right) - \frac{\alpha_{20}}{\alpha_{10}+\alpha_{20}}\Gamma\left(\frac{A}{2}\right) \qquad (27)$$

This is done by solving equation $\xi p'' + [(1-A) + A\xi^2]\, p' = 0$ with the three boundary conditions: $B_1(\bar{p}) = \bar{p}(\xi_0) = -\alpha_{20}$, $\xi_0 < \xi < \xi_F$; $B_2(\bar{p}) = \bar{p}(\xi_F) = 0$ *and* $\bar{p}(\infty) = \alpha_{10}$, *for* $\xi > \xi_F$

Use conditions B_1 and B_2 first and then analyse the condition at infinity [16].

Figure 2 shows $\bar{p}$ *vs* ξ and Figure 3 suggests that, as the initial relative fuel concentration increases, the flame is pushed away from the inner cylinder and vice-versa, when it goes down, the flame wraps around it. Figure 4 shows that if α_{10} and α_{20} are held constant, an increase in A will bring the flame towards the inner cylinder. Back to Equation (21) it might be interesting to notice that if

$Sc_i = Pr_i Le_i \cong 1$ then $A_i \approx \frac{\bar{a}_i}{\nu_{0i}} r_S^2 = \frac{(\bar{a}_i r_S) r_S}{\nu_{0i}}$, which is a Reynolds Number. Then it may be worth looking into the behavior of Equation (20) for values of $A_i \ll 1$, and $A_i = 1$. It can be easily shown that if $A_i \ll 1$, then the solutions for $\bar{p}$ will contain Ei functions (i.e., Exponential Integrals[15]) instead of

Gamma Functions. If $A_i = 1$ there will be Error Functions. In both instances they will keep the same structure as in Equations (24) and (25) as expected.

<u>Relative Species Concentrations and Temperature Profiles</u>

Recalling that $\alpha_i = a_i/\nu_i$, defining $\theta = c_p/Q$, taking $W = k_0\rho^2 a_1 a_2 \exp(-E/RT)$ and $\bar{E} = c_p E / RQ$, (Reference[8]), then Equations (5)-(7) can be written as:

$$-\frac{1}{2\xi}(\xi^2-1)\frac{d\alpha_i}{d\xi} = \frac{1}{Sc_i\eta_S^2}\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\alpha_i}{d\xi}\right) - Dm_i\alpha_1\alpha_2\exp\left[-\frac{\bar{E}}{\theta}\right] \qquad (28),(29)$$

and

$$-\frac{1}{2\xi}\left(\xi^2-1\right)\frac{d\theta}{d\xi} = \frac{\bar{a}_{i0}}{\eta_S^2 \Pr_i}\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\theta}{d\xi}\right) - Dm_i\alpha_1\alpha_2\exp\left(-\frac{\bar{E}}{\theta}\right) \qquad (30)$$

where the Dm_i are Damkohler Numbers defined as

$$Dm_i = \left(\frac{k_0}{\bar{a}i}\right)\nu\nu_2\left(\frac{\rho}{a_i}\right) \qquad (31)$$

<u>Concentration Profiles:</u>

If $\alpha_i = 0$, for i = 1 or 2 then θ is the solution of

$$-\frac{1}{2\xi}\left(\xi^2-1\right)\frac{d\theta}{d\xi} = \frac{\bar{a}_{i0}}{\eta_S^2 \Pr_i}\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\theta}{d\xi}\right) \qquad (31.a)$$

and α_j, $j \neq 1$, satisfies

$$-(\xi^2-1)\frac{d\alpha_j}{d\xi} = \frac{2}{\eta_S^2 Sc_i}\frac{d}{d\xi}\left(\alpha_j\frac{d\alpha_j}{d\xi}\right) \qquad (31.b)$$

Recalling that $A_i = Sc_i\,\eta_S^2\,/\,2$, then Eq. (31.b) can be written as

$$\xi\alpha_i'' + [(1-A_i) + A_i\xi^2]\alpha_i' = 0, \qquad (') = \frac{d}{d\xi} \qquad (32)$$

with the conditions
$\alpha_1(1_+)=0$, $\alpha_2(1_-)=0$, $\alpha_2(\xi_0)=\alpha_{20}$ and

$$\lim_{\xi\to\infty}\alpha_1 = \alpha_{10},$$

This yields

$$\alpha_1 = \alpha_{10}\frac{\gamma\left(\frac{A_1}{2},\frac{A_1}{2}\xi^2\right)-\gamma\left(\frac{A_1}{2},\frac{A_1}{2}\xi_F\right)}{\Gamma\left(\frac{A_1}{2}\right)-\gamma\left(\frac{A_1}{2},\frac{A_1}{2}\xi_F^2\right)}$$

$, \ \xi > \xi_F$

$$\alpha_1 = 0, \quad \xi_0 \le \xi \le \xi_F \qquad (33)$$

and

$$\alpha_2 = \alpha_{20}\frac{\gamma\left(\frac{A_2}{2},\frac{A_2}{2}\xi^2\right)-\gamma\left(\frac{A_2}{2},\frac{A_2}{2}\xi_F^2\right)}{\gamma\left(\frac{A_2}{2},\frac{A_2}{2}\xi_F^2\right)-\gamma\left(\frac{A_2}{2},\frac{A_2}{2}\xi_0^2\right)}$$

$$\xi_0 \le \xi \le \xi_F \qquad (34)$$

$$\alpha_2 = 0, \quad \xi > \xi_F$$

ξ_0 can be obtained from

$$\left.\frac{d\alpha_1}{d\xi}\right|_{\xi\to\xi_{F+}} = -\left.\frac{d\alpha_2}{d\xi}\right|_{\xi\to\xi_{F-}}$$

its solution yielding expression (26) for ξ_0, as before.

<u>Temperature Profiles</u>

Equation (31.a) can be written as

$$\xi\theta'' + [(1-B_i) + B_i\xi^2]\theta' = 0 \qquad i = 1,2 \qquad (35)$$

$$B_i = \frac{\eta_S^2 \Pr_i}{2\bar{a}_{i0}} \qquad (36)$$

Subjected to the conditions

$$i = 1,\ \theta(\xi_{F+}) = \theta_F,\ \theta(\infty) = \theta_{10} \qquad (37)$$
$$i = 2,\ \theta(\xi_{F-}) = \theta_F,\ \theta(\xi_0) = \theta_{20} \qquad (38)$$

yielding

$$\theta = \left(\theta_{10}-\theta_F\right)\frac{\gamma\left(\frac{B_1}{2},\frac{B_1}{2}\xi^2\right)-\gamma\left(\frac{B_1}{2},\frac{B_1}{2}\xi_F^2\right)}{\Gamma\left(\frac{B_1}{2}\right)-\gamma\left(\frac{B_1}{2},\frac{B_1}{2}\xi_F^2\right)}+\theta_F \qquad (39)$$

$, \ \xi > \xi_F$

$$\theta = (\theta_F - \theta_{20})\frac{\gamma\left(\frac{B_2}{2},\frac{B_2}{2}\xi^2\right)-\gamma\left(\frac{B_2}{2},\frac{B_2}{2}\xi_0^2\right)}{\gamma\left(\frac{B_2}{2},\frac{B_2}{2}\xi_F^2\right)-\gamma\left(\frac{B_2}{2},\frac{B_2}{2}\xi_0^2\right)}+$$

$$+\theta_{20}, \quad \xi_0 < \xi < \xi_F \tag{40}$$

It has been assumed that $\rho_{0i}= \rho$ on either side of the flame (this is a consequence of defining η through Equation (13) instead of taking it as

$$\eta^2 = (-1)^i\left(\frac{2\bar{a}_i}{\nu_{i0}}\right)\int_{r_i}^{r}\frac{\rho}{\rho_{i0}}r'dr'$$

as shown in Reference [1]. Then, if Lewis Numbers on both sides of the flame sheet are unity , i.e., $Le_i = 1$, and if $A_i = B_i$ $(i = 1,2)$, equations (28), (29) and (30) yield [8]:

$$-L_1(\theta) = L_1(\alpha_1) = \alpha_1\alpha_2 Dm_1 \exp(-\bar{E}/\theta) \tag{41}$$

and

$$-L_2(\theta) = L_2(\alpha_2) = \alpha_1\alpha_2 Dm_2 \exp(-\bar{E}/\theta) \tag{42}$$

where L is the differential operator [8]

$$L_i = \frac{1}{\Pr_i \eta_F^2}\frac{d^2}{d\xi^2} + \frac{1}{\xi}\left[\frac{1}{2}(\xi^2 - 1) + \frac{1}{\Pr_i \eta_F^2}\right]\frac{d}{d\xi} \tag{43}$$

Then

$$L_i(\theta+\alpha_i) = 0, \quad i=1, 2 \tag{44}$$

The discontinuity in the heat flux at the combustion surface can be found by integrating (44) across the reaction surface[8] so that

$$\left.\frac{d\theta}{d\xi}\right|_{\xi\to\xi_{F+}} - \left.\frac{d\theta}{d\xi}\right|_{\xi\to\xi_{F-}} = \left.\frac{d\alpha_1}{d\xi}\right|_{\xi\to\xi_{F+}} = \left.\frac{d\alpha_2}{d\xi}\right|_{\xi\to\xi_{F-}} \tag{45}$$

then, if $A_1 = A_2$,

$$\theta_F = \frac{\alpha_{20}}{\alpha_{10}+\alpha_{20}}\theta_{10} + \frac{\alpha_{10}}{\alpha_{10}+\alpha_{20}}\theta_{20} + \frac{\alpha_{10}+\alpha_{20}}{\alpha_{10}+\alpha_{20}} \tag{46}$$

If the fuel and the oxidant have the same initial dimensionless temperatures, i.e., if $\theta_{10}=\theta_{20}=\theta_0$ then

$$\theta_F = \theta_0 + \frac{\alpha_{10}\alpha_{20}}{\alpha_{10}+\alpha_{20}} \tag{47}$$

as expected [8].

Discussion of Results and Conclusions

This problem was examined following a "Burke Schumann" kind of approach, i.e., mass concentrations, temperature profiles and fuel consumption were calculated considering the instance of a combustion sheet i.e., of an infinitely thin reaction zone which is expected to occur when $Dm_i \to \infty$. It was also assumed that on either side of the flame an incompressible flow situation prevailed and that the flow field was known in advance so that, in a first approximation, $v_r(r)$ was independent of the temperature near the reaction zone. This is acceptable for low Mach Number flows and, besides, it leads to zeroth order approximation results of matched asymptotic analysis which might also be done. It is known [3, 15] that as the oxidant and the fuel enter the reaction zone after being heated by the heat flux from that zone, the reaction rate can be assumed to be high. This high reaction rate plus the limitation imposed on the mass rate of material consumed by the amount of gases being supplied, lead to small width of the reaction zone and to small fuel and oxidant concentrations within it. In the limiting case of an infinitely fast reaction this reaction zone becomes a geometric surface where the fuel and oxidant concentrations are zero. This is correct if the reaction rate is much faster then the rate of diffusion of the reactants and it is asymptotically correct as the ratio between the characteristic times of reaction and diffusion goes to zero [3, 15]. A consequence of this model is that the reactant fluxes entering the flame are in the stoichiometric ratio[3]. As expected, the flame temperature calculated choosing $Le_i = 1$, $i = 1,2$, assuming no heat losses, yielded the combustion temperature for the stoichiometric fuel/oxidant mixture ratio[3]. However, if Dm_1 and Dm_2 are finite, leading to the occurrence of a reaction zone of finite thickness whose structure should be investigated, then aconventional asymptotic technique could be used. In particular, if $Dm_i >> 1$ the temperature and concentration distributions in the combustion surface approximation are the first order terms of the asymptotic expansions in $1/Dm_i$ in the outer regions. If the flame structure is to be studied, then it is proper to follow Williams's suggestion of introducing a coupling function β and a mixture fraction Z, choosing then the latter and the Temperature T to be the main variables[15]. Finally notice that the proper stretching factor for this problem becomes Liñán's stretching variable in his classical counter flow problem[10], if the parameters A_i defined in Equation (21) are taken to be equal to one. This is shown in Appendix .

Acknowledgement

The author is indebted to Profs. P.A. Libby and F.A. Williams for suggesting the problem and for their invaluable suggestions and constant motivation. He also thanks Prof. A.M. Liñán for the fruitful discussions. This work was done under of the sponsorship of the Brazilian National Research Council, CNPq, project numbers 200751/90-5 and 500140/92-8 (NV).

References

1. Libby, P.A., Peters, N. and Williams, F.A., Combust. Flame. 75:265-280(1989).

2. Libby, P.A. and Williams, F.A., Combust. Sci. Technol. 31:1-42 (1983).

3. Libby, P.A. and Williams, F.A., Combust. Sci. Technol. 37:221-252 (1984).

4. Takeno, T. and Ishizuka, S., Combust. Flame 64:83-98 (1986).

5. Mikolaitis, D.W., Combust. Flame 56:327-335 (1984).

6. Takeno, T., Nishioka, M. and Ishizuka, S., Combust. Flame 66:271-283(1986).

7. Burke, S.P. and Schumann, T.E., Ind. Eng. Chem. 20:998-(1928).

8. Zel'dovich, Ya. B., Barenblatt, G.I., Librovich, V. B. and Makhviladze, The Mathematical Theory of Combustion and Explosions, Consultant Bureau, Plenum Publishing Co., New York, 1985, pp 555-580.

9. Spalding, D.B., ARS J. 31:763-771 (1961).

10. Liñán, A., Acta Astronautica 1:1007-1039

11. Peskin, R.L. and Wise, H., AIAA J. 4:1646-1650 (1966).

12. Kansas, D.R. and Williams, F.A., Phys. of Fluids, 12:265-267 (1969).

13. Buckmaster, J.D. and Ludford, G.S.S., Theory of Laminar Flames, Cambridge University Press, Cambridge, 1982, pp. 95 and 122.

14. Williams, F.A., Combustion Theory, Addison-Wesley, New York, 1988 (2nd Ed.), pp 10-18 and 73-84.

15. Abramowitz, M. and Stegun, I.A., Handbook of Mathematical Functions, Dover

16. Sinay, L., Lecture Notes, INPE/LCP, 1992 (In Portuguese).

Appendix

Note on the Coupling Functions β and Z for the problem.

Define β and Z as in Reference[14]:

$$\beta=\alpha_0-\alpha_F \tag{A-1}$$

$$Z=\frac{\beta-\beta_{O,0}}{\beta_{F,0}-\beta_{O,0}} \tag{A-2}$$

where $\alpha_F = -a_2/w_2\nu_2$, $\alpha_0 = -a_1/w_1\ \nu_1$, ($w_i$ being the molecular weight of species i), $\beta_{F,0} = -\alpha_{F,0} = -\alpha_{20}/w_2$ and $\beta_{O,0} = +\alpha_{O,0} = -\alpha_{10}/w_1$, so that Z = 1 in the fuel stream and Z=0 in the oxidant stream. Both Z and β are conserved parameters, i.e., $L_1(\beta) = L_2(\beta) = L_1(Z) = L_2(Z) = 0$. Following the earlier procedure and assumptions one can write

$$-(\xi^2-1)\frac{dZ}{d\xi}=A_i^{-1}\frac{d}{d\xi}(\xi\frac{dZ}{d\xi}) \tag{A-3}$$

where $A_i = \eta_F Sc_i/2$, i = 1,2 is to be solved with the boundary conditions

$$Z(\xi_{F+})=Z_c\ ,\ Z(\infty)=0\ ,\ \ i=1 \tag{A-4}$$
$$Z(\xi_{F-})=Z_c\ ,\ Z(\xi_0)=0\ ,\ \ i=2. \tag{A-5}$$

The above yields solutions of the type of

$$Z = a + b\,\gamma[A_i/2, (A_i/2)\xi^2], \tag{A-6}$$

where a and b are constants, as it can be easily seen from Equations (39) and (40) or (33) and (34). Then it is obvious that if one chooses Liñán's technique[10] to study the flame structure by doing the Activation Energy Asymptotics Method then the proper stretching variables for this problem would be

$$X = \gamma[A_i/2, (A_i/2)\xi^2], \quad i = 1,2 \tag{A-7}$$

Notice that if $A_i << 1$, these stretching variables become

$$X = Ei\,[(A_i/2)\xi^2] \tag{A-8}$$

and if $A_1 = A_2 = 1$ then

$$X = erfc[\xi / \sqrt{2}] \tag{A-9}$$

which is Liñán's stretching variable for the counter flow problem[10].

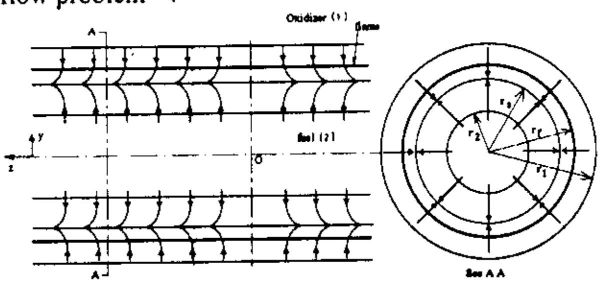

Figure 1: Problem Geometry

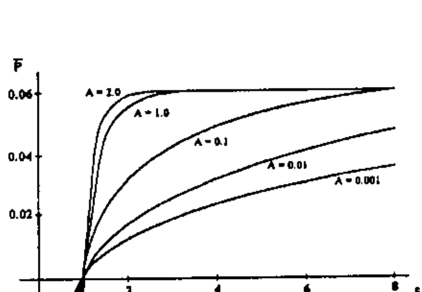

Figure 2: $\bar{P} = \alpha_1 - \alpha_2$, $\alpha_{10} = 0.03$, $\alpha_{20} = 0.06$, $A = A_i$

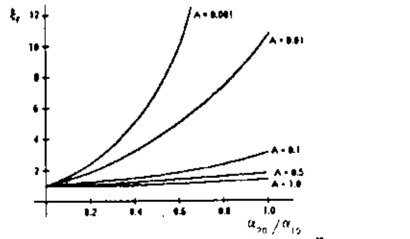

Figure 3: Flame Position, ξ_F, vs Initial Fuel Concentration, α_{20}/α_{10} For $A = A_i$

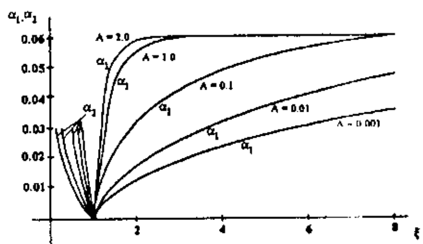

Figure 4: Concentrations, α_i, vs ξ
$\alpha_{10} = .06$, $\alpha_{20} = .03$