Interaction of a bouncing ball with a sinusoidally vibrating table

Elbert Macau[1], Marcus V. Carneiro[2], & Joaquim J. Barroso[3]

[1] Computing and Applied Mathematics Laboratory / National Institute for Space Research (INPE) / 12227-010 - Sao Jose dos Campos - SP - Brazil
[2] Swiss Federal Institute of Technology (ETH-Zürich)
[3] Associated Plasma Laboratory / National Institute for Space Research (INPE) / 12227-010 - Sao Jose dos Campos - SP - Brazil
elbert@lac.inpe.br

Exploring all its ramifications, this presentation gives an overview of the simple yet fundamental bouncing ball problem, which consists of a ball bouncing vertically on a sinusoidally vibrating table under the action of gravity. The dynamics is modeled on the basis of a discrete map of difference equations, which numerically solved fully reveals a rich variety of nonlinear behaviors, encompassing irregular non-periodic orbits, subharmonic and chaotic motions, chattering mechanisms, and also unbounded non-periodic orbits. For periodic motions, the corresponding conditions for stability and bifurcation are determined from analytical considerations of a reduced map. Through numerical examples, it is shown that a slight change in the initial conditions makes the ball motion switch from periodic to chaotic orbits bounded by a velocity strip $v = \pm \Gamma/(1-\epsilon)$, where Γ is the non-dimensionalized shaking acceleration and e the coefficient of restitution which quantifies the amount of energy lost in the ball-table collision. Moreover, a detailed numerical discussion of the excitation of the unstable 1-periodic mode and the ensuing transition to its stable counterpart mode is also given.