Adaptive multiresolution methods for 3D Euler equations: a case study

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Abstract

Discretization schemes in computational fluid dynamics can be accelerated significantly by adaptive multiresolution strategies without deteriorating the accuracy of the approximation [1, 2]. Starting with a reference explicit finite volume discretization, the goal of the present work consists in performing such a finite volume model in a more economic fashion, by taking into account local regularity information about the numerical solution. The main idea of multiresolution methods is to balance the truncation error of the underlying scheme on the finest regular grid and the threshold error introduced when discarding small detail coefficients of the multiresolution decomposition of the data.

At each time step, the multiresolution representation of the solution corresponds to the cell averages on a locally refined grid, which is expected to be coarse in regions where the solution is smooth, and refined in regions of sharp variations. To evolve the evolution in time, three basic operations are undertaken.

Since the regions of smoothness or irregularities of the solution may change with time, the current sparse grid may not be convenient anymore at the next time step. Therefore, a refinement operation is necessary to account for possible translation or creation of finer scales in the solution between two subsequent time steps. Then, the time evolution operator can be applied on the refined grid to advance the cell-averages. The corresponding adaptive flux computations are adopted at interfaces of cells, which may have different scales. To ensure conservativity of the scheme special care has to be taken at interfaces with changing. Finally, a thresholding operation is applied in order to unrefined those cells which are unnecessary for an accurate representation of the solution.

The implementation of the multiresolution representation uses a dynamic tree data structure, which allows to compress data, while still being able to navigate through it.

Examples of fully adaptive computations will be given for the compressible Euler equations in two and three space dimensions. The gain in memory and CPU time requirements with respect to the finite volume scheme on the corresponding finest grid will be discussed together with an assessment of the error.

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References

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