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CHARACTERIZATION OF HYPERCHAOTIC STATES IN PARAMETER-SPACE

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Historically, hyperchaos was first presented by Rössler [1] to characterize a chaotic system with more than one positive Lyapunov exponent. It means that the dynamics of the system is expanded in two or more directions simultaneously, resulting in a more complex chaotic attractor when we compare with the chaotic system with only one positive Lyapunov exponent. This expansion of the dynamics in two or more directions, makes hyperchaotic systems have better performance than chaotic systems in many chaos based fields, For example, hyperchaotic systems may be used to improve the security in chaotic communication systems, where a chaotic signal is used to mask the message to be transmitted, once messages masked by chaotic systems are not always secure [2].

In this paper we propose a method to numerically characterize the points with hyperchaotic behavior in two-dimensional parameter-spaces of dynamical systems modeled by a set of four nonlinear autonomous first order differential equations, which considers the magnitude of the second largest Lyapunov exponent. Each point of the parameter-space is painted with a color that indicates the level of hyperchaos of the point. Here we report specific results obtained for a prototype, which is a particular four-dimensional system constructed by us from a three-dimensional set of nonlinear autonomous first order differential equations proposed by Wang [3], by introducing a state feedback control to the first equation. The result is the new controlled system, given by

$$\begin{aligned} \dot{x} &= a(x-y) - yz + w, \\ \dot{y} &= -by + xz, \\ \dot{z} &= -cz + dx + xy, \\ \dot{w} &= -e(x+y), \end{aligned} \tag{1}$$

where x, y, z, w represent dynamical variables, and $a > 0$, $b > 0$, $c > 0$, $d > 0$, and $e > 0$ are parameters. Here the parameters $b = 9$, $c = 5$, and $d = 0.06$ are kept fixed, while a and e are simultaneously varied.

Figure 1 shows two parameter-space plots displaying different dynamical behaviors for system (1), both obtained by computing Lyapunov exponents on a 500×500 mesh of parameters (e,a). In Fig. 1(a) is considered the largest Lyapunov exponent, while Fig. 1(b) considers the second largest Lyapunov exponent. In these and further plots, system (1) was always integrated with a fourth-order Runge-Kutta algorithm with a fixed step size equal to 10^{-2}, and considering 5×10^5 steps to compute each exponent. Furthermore, with respect to the initial conditions, every orbit in the phase-space was started from the same (x_0,y_0,z_0,w_0)=(5,5,-5,-5), that is, we do not follow the attractor. Colors are associated with the magnitude of the Lyapunov exponent. White for more negative, black for zero, and red for more positive. Indeed, a positive exponent is indicated by a continuously changing yellow-red scale.

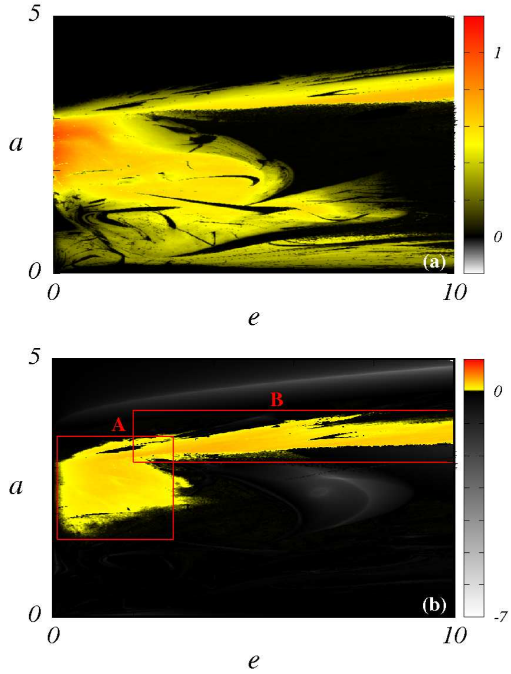

Figure 1 – Regions of different dynamical behaviors in (e,a) parameter-space of system (1), for $b = 9$, $c = 5$, and $d = 0.06$ (see text). (a) The largest Lyapunov exponent. (b) The second largest Lyapunov exponent.

As is well known, a negative largest Lyapunov exponent indicates a stable equilibrium point, a zero largest Lyapunov exponent indicates a stable periodic attractor (or quasiperiodic attractor, when the second largest Lyapunov exponent is also equal to zero), and a chaotic attractor has a positive largest Lyapunov exponent. Hereafter we concentrate our attention in the last case. In consequence, there are only two possibilities to the second largest Lyapunov exponent: a positive value, which indicates hyperchaotic motion, and a zero value indicating chaotic motion. Therefore, from above said, we conclude that the points pertaining to the yellow-red region in parameter-space of Fig. 1(b), are characterized for hyperchaotic motion. In other words, we completely numerically characterize hyperchaotic behavior in the parameter-space of the set of four nonlinear autonomous first order differential equations (1), by looking for the positive second largest Lyapunov exponent. For all set of parameters (e,a) that this exponent be greater than zero, the motion is hyperchaotic. Note that the yellow-red region in Fig. 1(b) is a subset of the region with same shading in Fig. 1(a). It is a consequence of the fact that if the second largest Lyapunov exponent is greater than zero, then the largest Lyapunov exponent also is greater than zero.

The structure of the hyperchaotic region can be better observed in a finer scale, as shown in Fig. 2 where appear magnifications of the boxes **A** and **B** in Fig. 1(b). Both plots

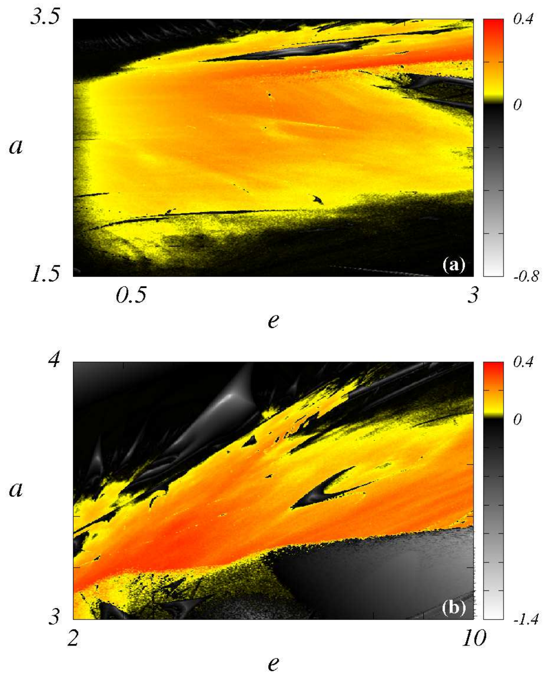

Figure 2 – Regions of different dynamical behaviors in (e,a) parameter-space of system (1), for $b = 9$, $c = 5$, and $d = 0.06$ (see text). (a) Magnification of the boxed region A in Fig. 1(b). (b) Magnification of the boxed region B in Fig. 1(b).

were obtained by computing the second largest Lyapunov exponent for each point in a 500 $\times$ 500 mesh of parameters (e,a). It is important to note that is possible to walk in the parameter-space, from one place to other place, by choosing adequate paths, that is, by producing adequate variations in parameter values it is possible to move along domains which are interesting for some practical application. For instance, it is possible to choose paths along which the second largest Lyapunov exponent is always positive, corresponding to yellow-red region in plots of Fig. 2, that is, paths that avoid regular regions for which the largest Lyapunov exponent is null, and the second largest is null or less than zero. Such paths are interesting for hyperchaotic communication systems.

This paper proposes the use of the second largest Lyapunov exponent as a measure of hyperchaotic motion, to construct two-dimensional parameter-space color plots for systems modeled by a set of at least four nonlinear autonomous first order differential equations. The method consists in to associate colors to the magnitude of the above-mentioned second largest Lyapunov exponent. More specifically, we use a continuously changing yellow-red scale to indicate a positive second largest Lyapunov exponent. The proposal for parameter-space hyperchaos visualization uses as a prototype a particular four-dimensional system with known equations.

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