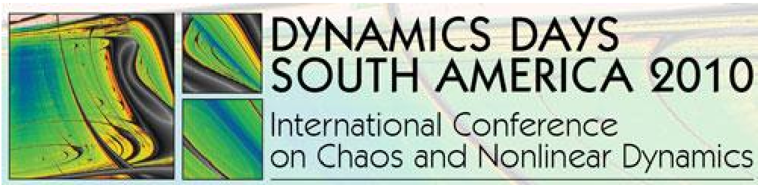

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LYAPUNOV EXPONENT DIAGRAM FOR A DRIVEN CHAOTIC OSCILLATOR WITH COMPLEX VARIABLE

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Chaotic phenomena can be observed in a diversity of systems, usually described by real variables [1]. The search of a real chaotic driven complex oscillator of the form $\dot{z} + f(z,\bar{z}) = \mathrm{A}e^{i\Omega t}$ is a challenge, where $f(z,\bar{z})$ is a quadratic polynomial function with real coefficients. Motivated by the recent work of Marshall and Sprott [2], we carried out a detailed numerical study of the following driven complex oscillator,

$$\dot{z} + (z-\bar{z})z + 1 = \mathrm{A}e^{i\Omega t}, \tag{1}$$

where $f(z,\bar{z}) = (z-\bar{z})z+1$. In Eq. (1), the complex variable z can be written as $z = x+iy$, with x, and y being real variables. $\bar{z}$ is the complex conjugate of z. From Eq. (1), we can derive an equivalent form to the driven two-dimensional system,

$$\begin{aligned} &\dot{x} - 2y^2 + 1 = \mathrm{A}\cos(\Omega t), \\ &\dot{y} + 2xy = \mathrm{A}\sin(\Omega t). \end{aligned} \tag{2}$$

Eqs. (2) is a nonautonomous two-dimensional system, which can be transformed to an autonomous three-dimensional system, Eqs. (3), with the transformation $w = \Omega t$.

$$\begin{aligned} &\dot{x} = 2y^2 - 1 + \mathrm{A}\cos(w), \\ &\dot{y} = -2xy + \mathrm{A}\sin(w), \\ &\dot{w} = \Omega. \end{aligned} \tag{3}$$

The numerical study carried out in this work consists of to calculate the largest Lyapunov exponent, numerically solving the Eqs. (3) with the fourth-order Runge-Kutta method with time step equal to 10^{-3}, for each pair of parameters (A,Ω). The range of parameter values was discretized in a mesh of 500×500 points equally spaced. We identify for each largest Lyapunov exponent a color, varying continuously from black (zero exponent), passing through yellow (positive exponent), up to red (positive exponent).

Fig. 1 shows the colorful Lyapunov exponent diagram for the parameters (A,Ω) of Eqs. (3). The color scale in right side is used to codify the largest Lyapunov exponent values in colors. The black regions represent periodic or quasi-periodic (2-tori) behaviors, and the yellowish and reddish regions represent chaotic behaviors. Inside the chaotic regions, we can observe the existence of immersed periodic structures, represented by the black regions inside of the yellowish and reddish regions. The green line in Fig.1 refers to the point positions studied in Ref. [2] for the Eqs. (3). Then, in our work we extended the regions studied in Ref. [2], given us a more detailed behavior of Eqs. (3).

Fig. 2 shows the phase portraits (attractors) located in the numbered green symbols of Fig. 1. All the phase portraits are in periodic regions, black colors in Fig. 1. In Fig. 2, we observe attractors with periodic behaviors (limit cycles), for example, attractors 1, 2, 6, 7, 8, 10, and 11, and attractors with quasi-periodic behaviors (2-tori), for example, attractors 3, 4, 5, 9, and 12.

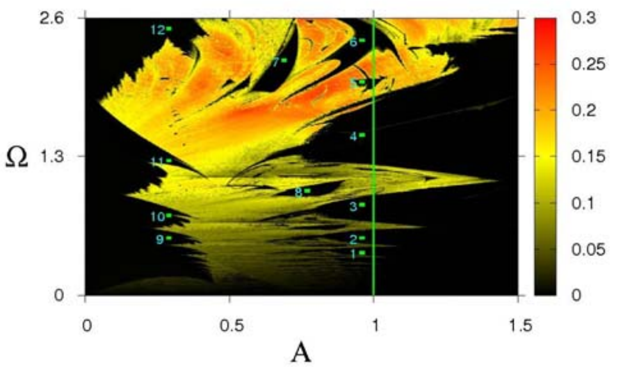

Figure 1 – Largest Lyapunov exponent diagram. The right side scale identifies in colors the largest exponent. The points along the green line in $\mathrm{A}=1$, locate the studies carried out in Ref. [2] for the Eqs. (3). The numbered green symbols localize the attractors shown in Fig. 2.

Periodic structures embedded in chaotic regions were reported in recent works [3-6], where the dynamical systems are modeled by a set of first-order differential equations. In those works, the observed periodic structures organize themselves in bifurcation cascades, called by period-adding cascades, that accumulate in periodic boundaries. That behavior seems to be a common feature presented in those systems. However, in the system studied here, modeled by Eqs. (3), that feature was not observed in Fig. 1.

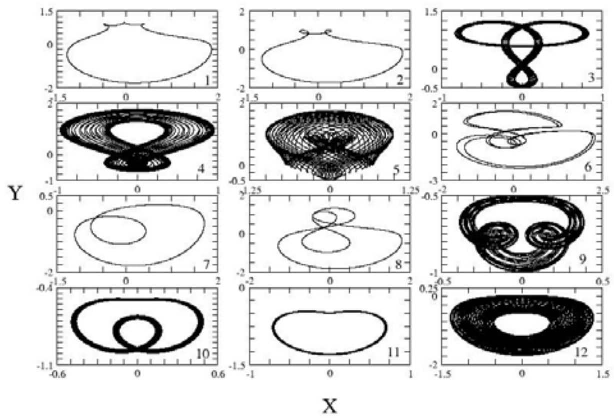

Figure 2 – Phase portraits of the numbered green symbols in Fig. 1. The inside numbers refer to the position of the attractors in Fig. 1.

A two-dimensional bifurcation diagram, using the largest Lyapunov exponent codified in a continuous range of colors, for a driven chaotic oscillator with complex variable was reported. We observed a diversity of periodic structures immersed in the chaotic regions. Periodic, quasi-periodic and chaotic behaviors were observed in the colorful diagram. The periodic behaviors were identified as limit cycles, the quasi-periodic ones were identified as 2-tori, and the chaotic ones were identified as chaotic attractors.

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