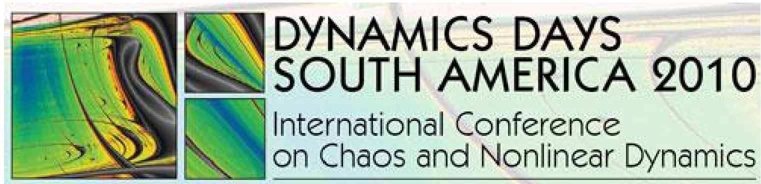

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COMPUTATION BY HETEROCLINIC SWITCHING

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How can nonlinear dynamical systems compute? Switching dynamics among saddles persistently emerges in a broad range of systems and may reliably encode "memory items" in neuronal networks cf. [1]. Their computational capabilities, however, are far from being understood. For models of neural networks, we here analyze how controllable persistent switching naturally emerges in the presence of inhomogeneities; we show that one system is capable of computing distinct binary functions, e.g. AND, OR, and XOR, by entering into switching sequences in a controlled way. These functions may be combined into any arbitrary logic function. Neural switching dynamics thus offers a highly flexible new kind of universal computation.

A heteroclinic connection in a dynamical system is a distinguished trajectory that links two saddles in state space. It occurs if unstable directions of one saddle are contained in the stable manifold of a second. A sequence of such connections linking several saddles cyclically is called a heteroclinic cycle. Several such heteroclinic cycles may emerge in a robust way provided the dynamical system considered exhibits a discrete symmetry. Their relations to symmetry make heteroclinic cycles of high current interest mathematically, but at the same time, their specific dynamical feature, supporting repetitive switching close to the saddles, pose an interesting challenge for the study of information encoding and computation, in particular in neuron-like systems [2–5].

For instance, it becomes more and more clear how information may be encoded by systems with heteroclinic cycles [6–11]. Recent studies even provided insights about how external perturbations may be processed [10, 12, 13], providing hints how such systems may actually compute using switching among saddles. Nevertheless, it is still unclear whether and how biologically inspired neural systems may actually perform generic computations using switching trajectories.

In this work, we show how symmetry-breaking external signals naturally induce switching dynamics close to what has been a heteroclinic cycle in a symmetric system of neuronal elements. We demonstrate this for fundamental forms of binary logic computations, in particular the XOR operation which provides a NOT operator and guarantees that the system may universally compute any desired function. Here, part of the signal inhomogeneity controls the kind of computational function (e.g. AND, OR, and XOR in our examples) and another part serves as the two-valued input to these functions. We present as a concrete example a network composed of five oscillatory spiking units, characterized by a single dynamical variable V, connected by symmetric delayed pulse-couplings. Such systems robustly exhibit a closed set of symmetry-related saddle states that are connected heteroclinically [10] and are capable of performing all basic bitwise operation over a pair of bits. Furthermore, as the signal-induced switching appears as a generic to feature of symmetry-breaking in systems with heteroclinic connections, the dynamics is insensitive to noise and can be found in a variety of coupled oscillating systems. These forms of neuronal switching dynamics thus provide a natural and flexible way of universal computation.

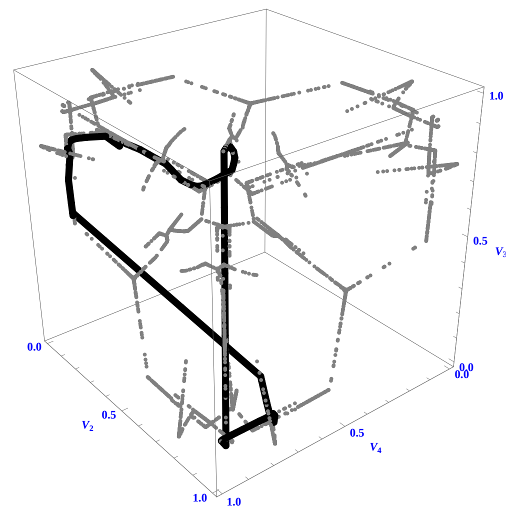

Figure 1 – Switching paths in a state space hyperplane for a neural network. Gray dots represent noise-induced paths; Black line represent a specific limit cycle generated by asymmetric current.

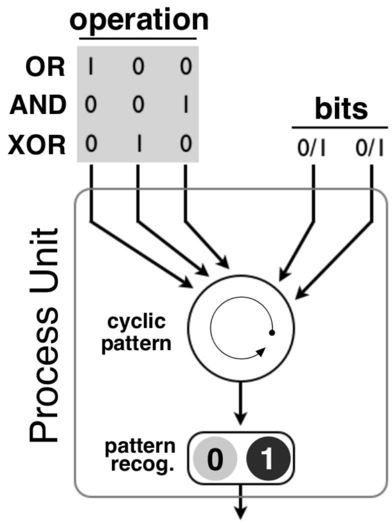

Figure 2 – Bitwise switching computer. Asymmetric external currents ($r_i \in \{0, 1\}$) generate specific limit cycles as internal representations. Categories of spike sequences (cyclic patterns) are associated to a zero or one pattern by the output layer.

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