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ON THE EFFECT OF A PARALLEL RESISTOR IN THE CHUA'S CIRCUIT

Flavio Prebianca[1], Gustavo Lambert[2], Holokx A. Albuquerque[3], Rero M. Rubinger[4]

1 Santa Catarina State University, Joinville, Brazil, flavio.prebianca@gmail.com
2 Santa Catarina State University, Joinville, Brazil, guto_33@yahoo.com.br
3 Santa Catarina State University, Joinville, Brazil, dfi2haa@joinville.udesc.br
4 Federal University of Itajubá, Itajubá, Brazil, rero@unifei.edu.br

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We studied the dynamical behavior of a Chua's circuit with parallel resistor. Our studies are based on a recent work of Braga, Mello, and Messias [1], where the authors analytically studied the governing nonlinear equations, where the nonlinearity of the diode is a cubic function, and the intrinsic inductor resistance was omitted. In our case, we numerically studied the complete set of equations, based on the Chua's circuit model [2], with a piecewise-linear function for the nonlinearity, and adding a resistor in parallel with the inductor. Our aim is to obtain the global bifurcation behaviors constructing two-dimensional parameter spaces of the model with the largest Lyapunov exponent method. We also realized the experimental circuit, and we obtained the experimental phase portraits (attractors) for three parameters of the system.

The set of equations that describes the Chua's circuit with parallel resistor is given by

$$\dot{v}_1 = \frac{dv_1}{dt} = (v_2 - v_1)/(RC_1) - i_d(v_1)/C_1 ,$$

$$\dot{v}_2 = \frac{dv_1}{dt} = (v_1 - \alpha v_2)/(RC_2) + i_L/C_2 , \qquad (1)$$

$$\dot{i}_L = \frac{di_L}{dt} = -v_2/L - i_L(r_L/L),$$

with the dynamical variables i_L, the current across the inductor, v_1 and v_2, the voltages across the capacitors C_1 and C_2, respectively. The nonlinearity is given by $i_d(v_1) = m_0 v_1 + \frac{1}{2}(m_1 - m_0)(|v_1 + b| - |v_1 - b|)$, and $\alpha = (R + R_P)/R_P$, where R_P is the parallel resistor which modifies the standard Chua's circuit, i.e., for $\alpha = 1$ [2].

The numerical study carried out in this work consists of to calculate the largest Lyapunov exponent, numerically solving the Eqs. (1) with fourth-order Runge-Kutta method with time step equal to 10^{-1}, for each pair of parameters (R, r_L). The range of parameter values was discretized in a mesh of 500×500 points equally spaced. We identify for each largest Lyapunov exponent a color, varying continuously from black (zero exponent), passing through yellow (positive exponent), up to red (positive exponent).

Figure 1 shows the two-dimensional parameter space for the parameters (R, r_L) in Eqs. (1). Black regions represent periodic behaviors, and the yellowish regions represent chaotic behaviors. The blue color represents the divergence of Eqs. (1). Inside the chaotic regions, we can observe the existence of immersed periodic structures, represented by the black regions inside of the yellowish regions. Figure 2 shows the attractors for parameter values in the two green marks of Fig. 1. We observe attractors with periodic (black regions) and chaotic behaviors (yellow regions).

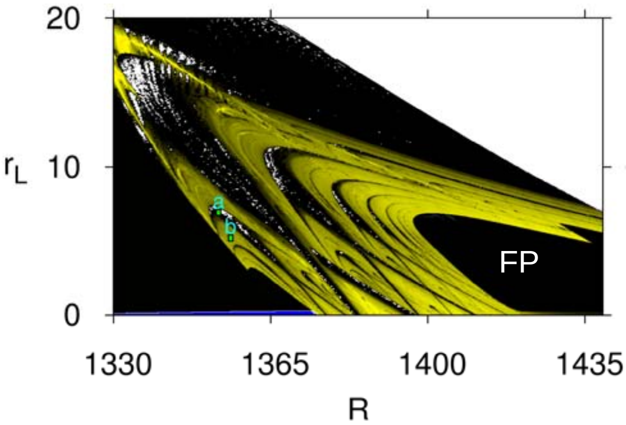

Figure 1 – Global view of the (R, r_L) parameter space of Eqs. (1). The axes are in resistance units (Ohms). Black color indicates periodic behavior, yellow one indicates chaotic behavior. The white region indicates fixed points. The blue region indicates divergence of Eqs. (1). The black region marked with FP, is another fixed points region. The points 'a' and 'b' locate the parameter values of the attractors shown in Fig. 2.

Periodic structures embedded in chaotic regions in the Chua's circuit were reported in recent theoretical and experimental works [2-5], where the Chua's circuit is modeled by Eqs. (1), for $\alpha = 1$, and with cubic nonlinearity [4] or piecewise-linear function [2,3,5]. In Ref. [5], we observed periodic structures organize themselves in a single spiral structure that coils up around a chaotic focal point in (R, r_L) parameter space, like that reported in Ref. [2]. However, in the Chua's circuit with parallel resistor here studied, i.e., Eqs. (1), the spiral structure was destroyed, and a large fixed points region emerged, shown in Fig. 1 as a large black region marked with FP.

In Fig. 3, we show four experimental attractors of the implemented Chua's circuit with parallel resistor. The experimental parameters were: (a) $R = 1362.0\,\Omega$, $r_L = 1.2$ Ω, $R_P = 2217.0$ Ω; (b) $R = 1375.0\,\Omega$, $r_L = 1.0$ Ω, $R_P = 2261.0$ Ω; (c) $R = 1384.0\,\Omega$, $r_L = 8.0$ Ω, $R_P = 2218.0$ Ω; (d) $R = 1434.0\,\Omega$, $r_L = 1.5$ Ω, $R_P = 2238.0$ Ω.

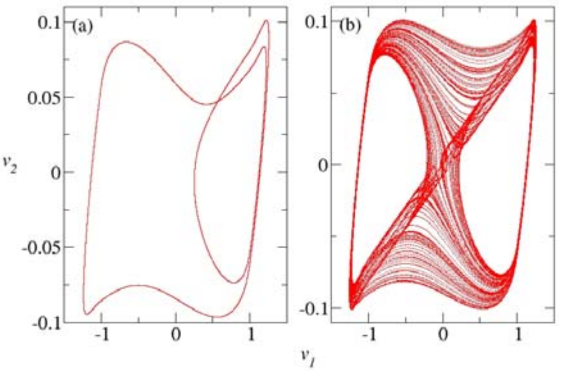

Figure 2 – Theoretical (a) periodic and (b) chaotic attractors for the parameter values of the two green marks 'a' and 'b' in Fig. 1. The axes are in voltage units (Volts).

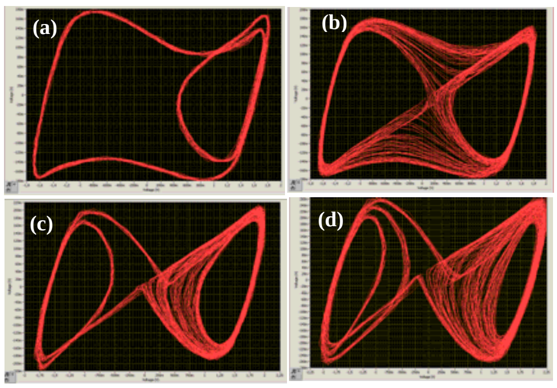

Figure 3 – Four experimental attractors for the implemented chaotic circuit. See the similarities between the attractors (a) and (b) with the theoretical attractors (a) and (b) in Fig. 2.

In Figs. 3(a) and 3(b), we can observe the similarities between the experimental attractors with the attractors obtained by numerical solutions of Eqs. (1) shown in Figs. 2(a) and 2(b). Two other experimental attractors, Figs. 3(c) and 3(d), were found in the implemented circuit. Those attractors present an interesting feature, periodic left side connected with chaotic right side.

A two-dimensional parameter space, using the largest Lyapunov exponent codified in a continuous range of colors, for the Chua's circuit with parallel resistor was reported. With that modification, we observed the disappearance of the spiral structure and the appearance of a fixed points region. In the experimental implementation of the circuit, we observed a good agreement between theoretical and experimental attractors. Two new experimental chaotic attractors were observed, with one side periodic and other one chaotic.

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