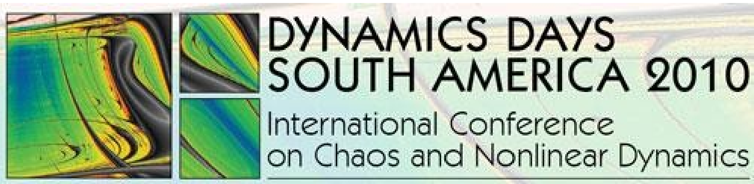

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Defects decay and pattern switching on 1-D Swift-Hohenberg equation

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The Swift-Hohenberg equation (SHE) is a well known model for pattern formation [1,2,3]. The non-linear differential equation was conceived as a representation of Rayleigh-Bérnard convection [3,4,5], but its form is recurrent in systems presenting patterns, especially stripe-like patterning, in various contexts. The basic SHE equation is:

$$\frac{\partial u}{\partial t}=[\varepsilon-(k_c+\nabla^2)^2]u-u^3 \qquad (1)$$

where, is ε a parameter related to the Rayleigh number, k_c is the critical wave vector, and u is the velocity vertical component of the fluid. We study the dynamics of defects in the one-dimensional SHE with periodic boundary conditions via numerical integration [12]. We define a defect as a local maximum on u^2 smaller than a threshold value (typically 90% of the mean of the 20% highest peaks), such that a pattern that has all peaks at similar heights would have no defect.

Simulating the dynamics for a random initial configuration of the fluid velocity, different values for the system length (L) and many different values for the parameter k_c, we observe a power-law decay of the number of defects with time, i.e., $\Delta N_D \propto t^{-\gamma}$, as shown in Figure 1. We use $\Delta N_D = N_D - N_\infty$, where N_∞ is the number of defects after a very long time relaxation, this may be different from zero for some values of the parameters, specially for $k_c \leq 0.4$. Surprisingly, the decay exponent γ depends linearly on k_c, this can be seen in Figure 2.

We also consider pattern switching by starting the system from a initial condition defined by a wave vector different from the one that grows more rapidly in the presence of noise.

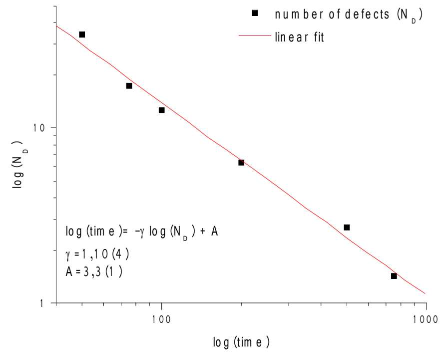

Figure 1 – Mean number of defects versus time in a set of 50 realizations, k_c=0.5 ; ε=0.1; L=500.

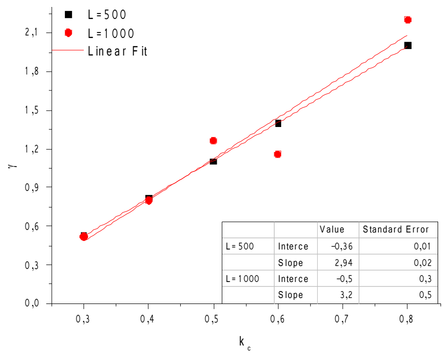

Figure 2 – Decay exponent γ versus k_c for L=500 and L=1000

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