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TRANSPORT PROPERTIES IN AN OPEN SYSTEM

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We have been studying transport properties of generic quantum open map that can exhibit integrable, chaotic or mixed dynamics in the classical limit. Although billiards would be physically more interesting, maps are, in general, mathematicaly much simpler.

Billiards are two-dimensional systems with a constant inner potential and hard wall confinement. Tecnological advances make possible to fabricate structures with a geometry similar to these billiards in two-dimensional electron gas (2DEG) which forms at the interface of a GaAs/AlGaAs heterostructure. Such electron billiards have dimensions L of the order of $1 - 10$ μm, while the Fermi wavelength $\lambda_F \approx 60$nm is much smaller than L. These are mesoscopic systems whose relevant dimensions are much smaller than the phase coherence length and the transport mean-free-path. In this ballistic regime semiclassical methods have been very successful in connecting quantum interference effects to the underlying classical dynamics [1].

In mesoscopic devices, in general, the confinement defines the classical dynamics to be regular (circle or square), chaotic (Bunimovich stadium or Sinai) or mixed. The main characteristics of the dynamics in mixed systems are bifurcations of periodic orbits, events in which different periodic orbits coalesce when parameters of the system are varied. In this situation, the Gutzwiller's trace formula fails. Then is necessary correct the trace formula for incorporate the bifurcations effects [2].

The main goal in this area is to study transport properties in bidimensional ballistic nanocavities at low electronic density and low temperatures. The understanding of quantum transport greatly owes to the Landauer-Buttiker approach of viewing conductance as a scattering problem [3]. The interpretation of conductance measurements in the ballistic regime from a quantum chaos point view involve the study of the quantum and classical mechanics of open systems [4]. Recently quantum open maps have been suggested as good models to simulate transport properties in chaotic systems. Open maps can model general properties of ballistic cavities coupled to electronic reservoirs [5].

One of the maps than can be recurrently found in the literature is the Chirikov map [5]. This map is defined as:

$$\begin{aligned} q_{t+1} &= q_t + p_t + \frac{\kappa}{4\pi}\sin(2\pi\, q_t) && (\bmod 1) \\ p_{t+1} &= p_t + \frac{\kappa}{4\pi}[\sin(2\pi\, q_t) + \sin(2\pi\, q_{t+1})] && (\bmod 1) \end{aligned} \tag{1}$$

where κ is the perturbation parameter of the system. For $\kappa = 0$ the system is integrable. For $\kappa > 0$ the system undergoes a transition to chaos. When $\kappa = 2\pi, 4\pi, 6\pi, 8\pi...$ fixed points of period bifurcate.

To model a pair of M-mode ballistic leads, we impose open boundary conditions in a subspace of Hilbert space represented by the indices n_m^α in coordinate representation. The subscript $m = 1, 2, \ldots, M$ labels the modes and the superscript $\alpha = 1, 2$ labels the leads. A $2M \times N$ projection matrix P describes the coupling to the ballistic leads. The elements of P are

$$P_{mn} = \begin{cases} 1 & \text{, if } n = m \in n_m^\alpha \\ 0 & \text{, otherwise} \end{cases}$$

Here, N denotes the Hilbert space dimension. Therefore the $N \times N$ matrix $Q = I - P^T P$ denotes all modes which are not lying on the leads, where I denote the unitary matrix. Particles are injected into the system by the leads, and at each iteration some of them leave the system while the remaining ones stay inside. Eventually all particles leave at the lead after a sufficient number of iterations. With this picture in mind, the $2M \times 2M$ scattering matrix is formed via a formal scattering series

$$S(\varepsilon) = P\frac{1}{[1-F]}FP^T \tag{2}$$

where $F = Ue^{i\varepsilon}$ is the quasienergy-dependent Floquet matrix of the closed system. The matrix S can be decomposed into 4 sub-blocks containing the $N \times N$ transmission and reflection matrices

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

where we can define the transmission ($T = Tr(tt^\dagger)$) e reflection ($R = Tr(rr^\dagger)$) coefficientes. A conductance G follows from the Landauer Formula [6]

$$G/G_0 \quad = \quad Tr(tt^\dagger) \tag{3}$$

where $G_0 = e^2/h$.

After removing vertical strips from phase space one gets the open map. In all the cases illustrated in this work, the ratio M/N of the dimension of the quantum map to the number of open channels is 0.28.

In order to see whether there are any effect of bifurcating fixed points of short period to the transmission T and reflection R, we have computed the scaterring matrix S in two different configurations as a function of the parameter κ. First, two strips are located in such a way that they block the bifurcating points: $q = 0.25$ and $q = 0.75$. Second, the two strips are placed such that they block the isolated points $q = 0$ and $q = 0.5$. These are period-1 fixed points. So, the two strips block the bifurcating (isolated) points in fig. 1a) (fig. 1b)).

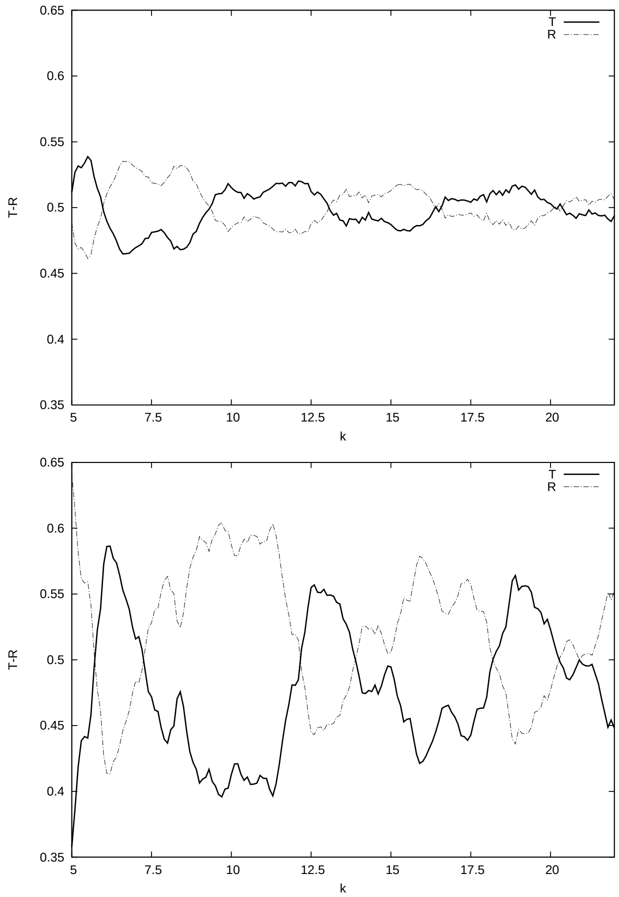

Figure 1 – Transmission T (continuous line) and Reflection R (dashed-dot line). Strips centred at the a) bifurcating and b) isolated fixed points.

The transmission (continuous line) and reflection (dashed-dot line) are plotted as a function of the perturbation parameter κ figures 1a), 1b). So far, the numerical results seem to show that there are strong oscillations in the transmission (and consequently in the reflectance) when the fixed points bifurcate.

This is a work in progress. We hope to be able to find some mathematical relations between the conductance G and the trace formula for the bounded map, like the connection between the Wigner time delay and the density matrix obtained in [7].

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