DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

DESCRIPTION OF REALISTIC WEALTH DISTRIBUTIONS BY KINETIC TRADING MODELS

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keywords: Applications of Nonlinear Sciences; Collective Phenomena; Complex Networks

Economic systems share features common to a wide variety of complex systems [1]. In particular, the problem of wealth distribution has been dealt with a number of physics-based models which, while simple, reproduce various features of economic systems.

Example of these are the kinetic trading models [2, 3], that are represented by a number of agents, which interact by trading money. Total money is conserved, and wealth is distributed across agents, eventually reaching an equilibrium distribution which depends on the details of the interaction. Thus, these models are analogous to a simulation of interacting particles in a gas where agents that trade and exchange money correspond to particles that collide and exchange energy.

In this paper we show that a simple kinetic model for trading is able to reproduce observed wealth distribution data. For economic systems, it is a well known observation that the probability of an agent of having wealth x is

$$P(x) \propto x^{-(1+\alpha)} \tag{1}$$

for large x, with an observed value of α between 1 and 2. This was first noted by Pareto in the 1890's [4], and has been also observed in different countries and in different periods of time.

In a kinetic model, if agents can exchange any amount of their current wealth, a Maxwellian equilibrium is found. A power law tail can be obtained in more refined models, assigning a random saving propensity $0 \leq \lambda_i < 1$ to each agent, such that, at each interaction each agent only trades a certain amount of her/his wealth. However, these models yield power law tails with exponent 1 [5], or are restricted to a particular choice of the distribution of λ_i, or to only fit the asymptotic behavior, in the Pareto regime [2, 3, 6, 7]. We will show, instead, that kinetic trading models are able to fit observed wealth distribution data not only for Pareto indexes $\alpha \neq 1$, but also for all wealth ranges. Thus, these models can be quantitatively, not just qualitatively, consistent with observed data.

We start by reformulating the kinetic trading model mentioned above in terms of an spending propensity $0 < \Lambda_i \leq 1$ of each of the N agents. An agent i who has money $x_{i,t}$ at time t, exchanges part of her/his money with an agent j, such that at time $t+1$ their respective money is

$$x_{i,t+1} = (1-\Lambda_i)x_{i,t} + (\Lambda_i x_{i,t} + \Lambda_j x_{j,t})\,\epsilon_{i,j,t} \ , \tag{2}$$

$$x_{j,t+1} = (1-\Lambda_j)x_{j,t} + (\Lambda_i x_{i,t} + \Lambda_j x_{j,t})\,(1-\epsilon_{i,j,t}) \ , \tag{3}$$

where the case $j = i$ is never reached, as we do not consider self-interactions (an agent exchanging money with herself/himself). Here t only labels time steps, and $\epsilon_{i,j,t}$ is taken from a uniform distribution $U(0,1)$. [Notice that there is no sum over repeated indexes in (2)]

Equations (2) and (3) describe a single interaction at time t between two given agents i and j. The simulation is started by assigning to each agent a spending propensity $0 < \Lambda_i \leq 1$, from a given distribution as discussed above, and a certain amount of money $x_{i,0}$. The model is then iterated for a long enough time, choosing at random which pair of agents interact at each time step. In every respect, the procedure is the same as if a saving propensity λ_i is used instead, as mentioned above. The change $\lambda_i \to \Lambda_i = 1 - \lambda_i$ may appear as trivial, but it turns out that high end tails are more sensitive to non-uniformity in Λ_i rather than λ_i [3]. This is due, in turn, to the nontrivial mapping of distributions when they are nonuniform.

Starting from Eq. (2) it can be shown [8] that the average wealth (with respect to time) of agent i is related to the spending propensity as

$$\langle x_{i,t} \rangle = \frac{\kappa}{\Lambda_i} \ , \tag{4}$$

where κ is a constant, which can be calculated using the conservation of money, and that the wealth distribution $P(x)$ and the spending propensities distribution $P(\Lambda)$ are related by

$$P(x) = P(\Lambda) \Bigg|_{\Lambda=\frac{\kappa}{x}} \frac{\kappa}{x^2} \ . \tag{5}$$

For a uniform distribution of Λ, $P(\Lambda)$ constant, we recover the known result that the wealth distribution follows a power law with Pareto index $\alpha = 1$.

Equations (4) and (5) establish how the distribution in spending propensities $P(\Lambda)$ determines the wealth distribution $P(x)$. In fact, using them we can show that kinetic trading models can fit observed distributions for an arbitrary Pareto index α, and also for all wealth ranges, not only at the high end. This will be explicitly done by simulating actual wealth distribution data.

Here we show results for "The World Distribution of Household Wealth" published by The World Institute for Development Economics Research of the United Nations University (UNU-WIDER) [9]. As seen in Fig. 1, it shows a power law tail with index $\alpha \simeq 1.5$, consistent with the original observation by Pareto [4].

Data for the UNU-WIDER study are plotted as gray dots in Fig. 1(a). Simulation results, when using model (2), are presented in Figs. 1(a) and (b) for wealth and Λ distributions, respectively. It can be seen that this simple model of exchange with spending propensities is able to reproduce the observed data, even if the Pareto index is clearly different from 1. Furthermore, the model works over the whole range of wealth, not just the high-end tail.

Similar results have been obtained by analyzing the 2006 Forbes list of billionaires of the world, which better represent data for the high-end of the wealth distribution [8].

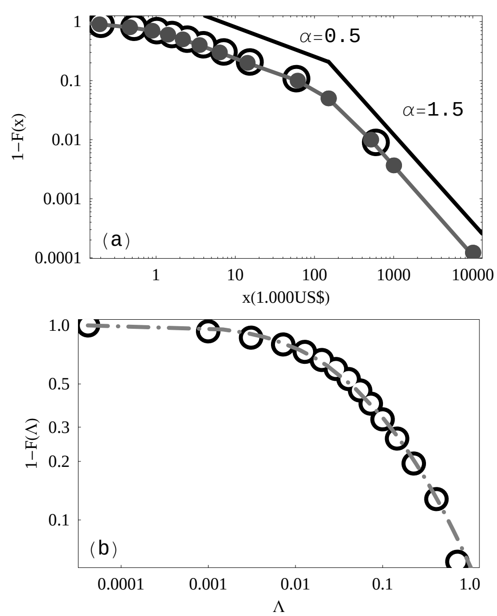

Figure 1 – Data and model fit for UNU-WIDER's data [9]. Rings: model results; dots: original data; light line: analytic results from Eq. (4); dark line: power-law fit. (a) Distribution function for wealth x, from UNU-WIDER's data. (b) Distribution function for spending propensity Λ, fitted by a log-normal distribution (dashed line). $F(x)$ and $F(\Lambda)$ denote cumulative distribution functions.

These findings also show that non-dissipative binary interactions between otherwise independent agents, lead to power law distributions, as long as full exchange is not possible (see [10] for a similar suggestion). Such an insight may be useful to describe other physical systems which tend toward non-Maxwellian equilibria.

We would like to stress here that the model provides also a spending propensity distribution, which can be regarded as a prediction of the model, which can in principle be tested against real data. Certainly, it would be interesting to validate the Λ distribution by independent means, but that requires to have reliable data for a given community, both for its spending propensity and wealth. We plan to pursue such line of research in a future work.

The current model has the restriction that interactions can occur between any pair of agents. However, in real economic systems, interactions may be limited by many environmental or psychological contraints like geographic location, nationality, language, educational level, etc. This might well be represented as economic interaction over a complex network, a study which is currently being carried out.

This project has been financially supported by FONDECyT under contracts 1070854 (JAV), 1070131 (JAV), 7070307 (JAV), 1070080 (JR), 1071062 (JR), 1060830 (VM), 1080658 (VM) and 3060029 (BT). We also acknowledge COLCIENCIAS's financial support to Ph. D. students (NL), and the Research Fund of the Engineering Faculty of the Universidad de los Andes, Colombia (RZ).

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