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PERIODIC WINDOWS DISTRIBUTION: A QUANTITATIVE DESCRIPTION IN THE TWO-PARAMETER SPACE

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For several smooth nonlinear maps and differential equations, stable periodic orbits and their dependence on the system control parameters are well known. The existence of these orbits can be properly visualized on a bi-dimensional parameter space, where generally we find *periodic windows*, i.e., continuous sets of parameters, embedded into chaotic regions, for which periodical orbits exist [1, 2]. There is an intricate periodic window, quite general in dynamical systems, whose local nature was explained in [1, 3] but the global features, despite the great number of studies [4–16], remain not well understood. For local and global features we mean typically qualities of an isolated and a multiple periodic windows, respectively. Recently, it has been found that these periodic windows, baptized shrimp [2], are continuously connected along spirals emerging from in a homoclinic bifurcation point [17, 18] (set of parameters for which a bi-asymptotic curve, the homoclinic orbit, converges to a saddle-focus equilibrium point).

However, in these researches the distribution of shrimps in the parameter space have not yet been clearly associated to any fundamental dynamical property. To accomplish this, we investigate the relation between the shrimps and the homoclinic curves in the parameter space. For the Rössler system, we present a new and remarkable two-parameter space scenery where from each shrimp emerge infinity spirals with focus in discrete points along of a homoclinic bifurcation curve (continuous parameter sets for which homoclinic orbits exist). Each spiral is composed by a shrimp family , *i.e.*, infinite shrimps continuously connected in a spiral sequence. We show that, even the shrimps are a codimension-two phenomena (two parameters are necessary to obtain it), they are accumulating at the spiral focus following scaling laws predicted by the one-parameter space homoclinic theory [19–22]. We also show that the reported period adding cascades observed in shrimps accumulations [11, 12] is a consequence of the spiral periodic windows approach to the homoclinic bifurcation point [23].

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