ALFVÉN PARAMETRIC INSTABILITIES IN SPACE PLASMAS

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Abstract

The parametric instabilities driven by a standing Alfvén wave of circular polarization are studied. Above a certain threshold amplitude, a standing Alfvén wave can generate convective or purely growing MHD parametric processes. It is shown that the threshold conditions can be satisfied by the ULF waves in the planetary magnetospheres. Large density fluctuations and cavities may result from the ponderomotive interaction of Alfvén and acoustic waves. Application of this theory to the observation of Alfvén-acoustic turbulence in the Earth's auroral plasma is discussed.

1. INTRODUCTION

Extensive observational data of standing Alfvén waves in the Earth's magnetosphere are now available [1,2]. Recently, Fukunishi & Lanzerotti [3] detected ground signature of standing Alfvén waves excited by the flux transfer events, indicating that these waves may provide key information on solar wind-magnetosphere coupling at the Earth's dayside magnetopause. Bloch & Fälthammar [4] reported the measurement by the Viking satellite of standing Alfvén waves in the 0.1-1 Hz frequency range trapped within a density cavity of high Alfvén velocity between about half and a few Earth radii; the observed wave spectra are usually quite broad, albeit almost monochromatic waves around 0.4-0.6 Hz are often seen in the late morning sector. Knudsen et al. [5] used rocket and HILAT satellite data to analyze the electric and magnetic fields of standing Alfvén waves in the Earth's auroral ionosphere.

Standing Alfvén waves in the planetary magnetospheres may sometimes be of finite amplitude (the meaning of "finite amplitude" will be clarified in Section 2). A finite amplitude Alfvén wave may be unstable to parametric instabilities, leading to the development of a turbulent cascade [6,7]. Previous theories of Alfvén parametric instabilities are mostly restricted to a traveling pump wave. The case of a standing pump wave was studied by Hung [8] and Lashmore-Davies & Ong [9], with a number of simplified assumptions. Their assumption of linear wave polarization is not applicable for noncompressible Alfvén waves; in addition their analysis is based on a normal mode formalism which is only valid for interactions involving resonant modes. In this paper, we present a new formulation of parametric instabilities generated by a standing (or traveling) circularly polarized Alfvén wave, which is valid even in strongly driven regimes wherein nonresonant acoustic modes are generated. Our treatment includes both the left-hand circularly polarized and right-hand circularly polarized MHD modes. In Section 2, the theory is given. In Section 3, we discuss some application of our theory to the planetary magnetospheres.

2. THEORY

Express the total magnetic field by $\mathbf{B} = B_0\hat{\mathbf{z}} + \mathbf{b}_0 + \mathbf{b}$, where $B_0\hat{\mathbf{z}}$ is the uniform ambient longitudinal magnetic field, $\mathbf{b}_0$ is the pump Alfvén field, and $\mathbf{b}$ is the induced transverse magnetic field. We assume that all the interacting waves travel along the ambient magnetic field. The pump field consists of two oppositely propagating right(left)-hand circularly polarized Alfvén waves, $\mathbf{b}_0 = \mathbf{b}_0^+ + \mathbf{b}_0^-$, with $\mathbf{b}_0^+(\mathbf{r},t) = \hat{\mathbf{e}}_\pm(b_0^+/2)\exp[i(k_0 z - \omega_0 t)] + c.c.$ and $\mathbf{b}_0^-(\mathbf{r},t) = \hat{\mathbf{e}}_\pm(b_0^-/2)\exp[i(-k_0 z - \omega_0 t)] + c.c.$, where b_0^+ and b_0^- are scalar amplitudes and the polarization unit vectors $\hat{\mathbf{e}}_\pm = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$ denote right(left)-hand circular polarization for the plus(minus) sign. A finite amplitude standing Alfvén wave can parametrically couple to fluctuactions in magnetic field $\mathbf{b}(\mathbf{r},t)$ and particle density $\rho(\mathbf{r},t) = (\rho/2)\exp[i(kz-\omega t)] + c.c.$. The beating of pump field with density fluctuations gives rise to various induced modes of magnetic fluctuations. The characteristics of the induced magnetic modes are determined by the type of wave coupling. In analogy with 3-wave parametric processes driven by a traveling pump we can identify two distinct types of wave coupling, depending on the wave vector kinematics, for MHD parametric processes driven by a standing pump [10,11,12]: decay-type ($b_0 \to b + \rho$) and fusion-type ($b_0 + \rho \to b$), where the acoustic mode ρ can be either resonant or nonresonant. In the presence of two counter propagating pump waves, two distinct low-frequency modes can be excited [12]: $\rho(\omega, \pm k)$ and $\hat{\rho}(\omega, \pm k \mp 2k_0)$ for the decay-type interaction. For simplicity, in this paper we assume $\hat{\rho}$ off-resonant and only consider the ρ mode. The role of $\hat{\rho}$ will be analysed in a future paper.

The set of coupled wave equations derived from MHD equations [7] that governs the parametric interaction of magnetic and density fluctuations is given by

$$(\partial_t^2 - c_A^2\partial_z^2)\mathbf{b} = -\frac{B_0}{\rho_0}\partial_z(\rho\partial_t\mathbf{v}_0) - \partial_{zt}^2(v_z\mathbf{b}_0) - B_0\partial_z(v_z\partial_z\mathbf{v}_0), \tag{1}$$

$$(\partial_t^2 - c_S^2\partial_z^2)\rho = \frac{1}{2\mu_0}\partial_z^2(\mathbf{b}_0\cdot\mathbf{b}), \tag{2}$$

where ρ_0 is the unperturbed particle density, $c_A = B_0/(\mu_0\rho_0)^{1/2}$ is the Alfvén velocity, $c_S = (P_0/\gamma\rho_0)^{1/2}$ is the acoustic velocity, $\mathbf{v}_0 = (\pm\mathbf{k}_0\cdot\mathbf{B}_0)\mathbf{b}_0/(\mu_0\rho_0)^{\frac{1}{2}}$, and v_z is related to ρ by $\partial_t\rho + \rho_0\partial_z v_z = 0$. The RHS of (1) is related to the current density arising from the beating of the pump Alfvén wave and density fluctuations. The RHS of (2) is related to the ponderomotive force resulting from the beating of the pump and induced Alfvén waves. Equation (1) shows that, in order to satisfy the conservation law of wave helicity [13], only magnetic fluctuations with the same sense of polarization as the pump field are excited. Since the plasma parameter $\beta \equiv c_S^2/c_A^2$ is normally much smaller than unity in the planetary magnetospheres [2], we shall take the limit $\beta \ll 1$ for the remainder of this paper. If $\beta \ll 1$, the last two terms in the RHS of (1) can be ignored.

The decay-type interaction is described by the coupling of the pump field with the Stokes Alfvén mode $\mathbf{b}_-^+(\omega_0 - \omega^*, \mathbf{k}_0 - \mathbf{k})$ induced by the forward pump, the anti-Stokes Alfvén mode $\mathbf{b}_+^-(\omega_0 + \omega, -\mathbf{k}_0 + \mathbf{k})$ induced by the backward pump, and the associated density fluctuations. The other pair of sidebands, $\mathbf{b}_+^+$ and $\mathbf{b}_-^-$, are considered off-resonant because of their large wave numbers. Assuming $b_0^\pm >> b_-^+$ and b_+^-, the Fourier-transform of (1) and (2) then yields

$$D_-^+ b_-^+ = -\frac{\omega_0^2 k_-^+}{2k_0\rho_0}\rho^* b_0^+, \tag{3}$$

$$D_+^- b_+^- = -\frac{\omega_0^2 k_-^+}{2k_0\rho_0}\rho b_0^-, \tag{4}$$

$$D\rho = \frac{k^2}{4\mu_0}(b_0^+ b_-^{+*} + b_0^{-*} b_+^-), \tag{5}$$

where $D_-^+(\omega_-^+, \mathbf{k}_-^+) = (\omega_0 - \omega^*)^2 - c_A^2(\mathbf{k}_0 - \mathbf{k})^2$, $D_+^-(\omega_+^-, \mathbf{k}_+^-) = (\omega_0 + \omega)^2 - c_A^2(\mathbf{k}_0 - \mathbf{k})^2$, $D(\omega, \mathbf{k}) = \omega^2 - c_S^2\mathbf{k}^2$ and we set $k_-^+ = -k_+^-$ in (4). Using the resonant approximation and considering a standing pump ($|b_0^+| = |b_0^-| \equiv b_0$) we get from (3) - (5)

$$(\omega^2 - c_S^2k^2)(\omega^2 - \delta^2) = \delta W, \tag{6}$$

where $\delta = \omega_A - \omega_0$ with $\omega_A = -c_A k_-^+$ and $W = (\omega_0^3 k^2 A)/(8k_0^2)$ with $A = b_0^2/B_0^2$. Observe that $W > 0$ always and that ω_A is the unpertubed fequency of magnetic fluctuations which is different from the pertubed frequencies $\omega_-^+ = \omega_0 - \omega^*$ and $\omega_+^- = \omega_0 + \omega$. Analogously, for the fusion tipe interactions, we have similar equations to (3) – (5) which yelds to (6) with $\omega_A = -c_A k_+^+$.

Let us analyse the dispersion relation (6), valid for both decay(fusion)-type couplings. Convective instabilities with $Re(\omega) \neq 0$ are excited if $\delta < 0$. In this case, the standing Alfvén wave pump generates a pair of counter-directed oscillatory acoustic waves, and a pair of counter-directed Alfvén waves $\mathbf{b}_+^-(\mathbf{b}_+^+)$ with upconverted frequency $\omega_0 + \omega$, and $\mathbf{b}_-^+(\mathbf{b}_-^-)$ with downconverted frequency $\omega_0 - \omega^*$. Two regimes of the convective instability can be distinguished. In the weak pump regime ($|\omega|^2 \simeq c_S^2 k^2$), a resonant instability is excited in which the acoustic waves are resonant (normal) modes having frequency near $c_S|k|$; the growth rate is $\Gamma = (W/4c_S|k|)^{1/2}$. In the strong pump regime ($|\omega|^2 \gg c_S^2 k^2$), a nonresonant instability is produced in which the acoustic waves are nonresonant (quasi-reactive) modes with frequency far from $c_S|k|$; the growth rate is $\Gamma = (3/4)^{1/2}(W/4)^{1/3}$. On the other hand, purely growing instabilities with zero-frequency acoustic fluctuations are generated if $\delta > 0$. In this case, the standing Alfvén pump excites a pair of counter-propagating Alfvén waves, both oscillating at the pump frequency ω_0, thus forming an induced standing Alfvén wave, via coupling to purely growing density fluctuations. This case can be called Alfvén oscillating two-stream instability, which can be divided into two regimes. In the subsonic regime ($\Gamma^2 \ll c_S^2 k^2$), the maximum growth rate is $\Gamma_{max} = |\delta_{max}| = W/(2c_S^2 k^2)$. In the supersonic regime ($\Gamma^2 \gg c_S^2 k^2$), the maximum growth rate is $\Gamma_{max} = |\delta_{max}| = (W/2)^{1/3}$. The nonresonant convective and supersonic purely growing instabilities occur when $W >> c_S^3 |k|^3$.

Even small, but finite-amplitude (i.e., $b_0/B_0 \neq 0$) Alfvén waves with $b_0/B_0 \ll 1$ are capable of generating MHD parametric instabilities provided their field values exceed a certain threshold which depends on the damping rates. The dissipative threshold for the resonant convective instability, obtained by including the damping rates in (6), is $W_T = c_S|k|\Gamma_S\Gamma_A$ which depends on the damping rates of both acoustic (Γ_S) and Alfvénic (Γ_A) waves; whereas, the minimum threshold for the subsonic oscillating two-stream instability, $W_T = c_S^2 k^2 \Gamma_A$, is independent of Γ_S. Since in general $\Gamma_S/c_S|k| \ll 1$ and $\Gamma_A/c_A|k| \ll 1$, the above threshold conditions can be satisfied by small but finite-amplitude ULF waves in the planetary magnetospheres, with $b_0/B_0 \sim 10^{-2} - 10^{-3}$.

3. DISCUSSION AND CONCLUSION

Finite amplitude Alfvén waves with electric field strengths greater than 100 mV/m were observed on recent sounding rocket flights in the Earth's auroral region [14,5]. Both standing and traveling Alfvén wave patterns were identified, depending whether the wave frequency is lower or higher than 1 Hz. The largest waveforms consist of step functions, rather than near-sinusoidal waves. Significant density perturbations with density cavity of order one were detected during the intense Alfvén event. These observations of Alfvén-acoustic turbulence are strongly suggestive of MHD parametric processes studied in Section 2. In fact, our theory renders support for the suggestion made by Boehm et al. [14] that the observed features are due to the ponderomotive effects associated with the parametric instabilities. The ponderomotive effects are evident in the RHS of (2) and (5), which result from the coupling of pump and induced Alfvén waves. This ponderomotive force acts on the acoustic wave to amplify the density perturbations. In the subsonic limit ($\partial_t^2 \ll c_S^2 \partial_z^2$), (2) yields the relation $\rho \propto -|\mathbf{b}|^2$, which shows that a density cavity appears in the region of high Alfvén wave amplitude. For an Alfvén wave amplitude of 150 nT and background density of $3 \times 10^4 cm^{-3}$ measured in the auroral plasma, the ponderomotive potential is 0.9 eV, which is much larger than the background temperature. Thus, the ponderomotive effects of the observed intense Alfvén waves may be the cause of large density variations. The observed density cavity is localized, indicating that the MHD parametric instability is operating in the purely growing regime. According to our theory, for the purely growing Alfvén oscilating two-stream instability, density perturbations grow at a rate $\Gamma_{max} \propto b_0^2$ in the subsonic regime and $\Gamma_{max} \propto b_0^{2/3}$ in the supersonic regime. Hence, the larger the pump amplitude, the larger the growth rate and consequently the larger the density fluctuations. This is in good agreement with the auroral rocket data [14], which show that for near-sinusoidal small-amplitude Alfvén waves $\rho/\rho_0 \ll 1$, whereas for step-like intense Alfvén waves $\rho/\rho_0 \rightarrow 1$.

The MHD parametric instabilities discussed in Section 2 may lead to turbulent dissipation process whereby the energy of Alfvén waves is converted to the kinetic energy of plasma particles. The conversion of wave energy to particle energy may be due, for example, to the acoustic waves induced by an MHD parametric instability being damped via Landau damping [15]. This dissipation mechanism resulting from wave-particle interactions may produce plasma heating as well as particle acceleration. In fact, during the Alfvén-acoustic turbulence event reported by [14], significant temperature increase and energetic electron precipitation were both observed in the auroral plasma.

In addition to the Earth's magnetosphere, finite amplitude standing Alfvén waves were detected by Voyager 1 in the Io plasma torus [16,17], as predicted by the nonlinear Alfvén wave current model of Gurnett & Goertz [18]. Presumably, the theory developed in Section 2 should have relevant applications in the magnetosphere of Jupiter and other planets.

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4. REFERENCES

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