NUMERICAL INVERSION OF THE LAPLACE TRANSFORM IN THE INITIALIZATION OF ATMOSPHERIC BAROTROPIC MODELS

Julio Cesar Ruiz Claeyssen
Haroldo Campos Velho
Instituto de Pesquisas Espaciais - INPE/MCT
Laboratório Associado de Computação e Matemática Aplicada
Caixa Postal 515, 12201, São José dos Campos, SP

Marco Tulio Vilhena
Escola de Engenharia - DEN
Universidade Federal do Rio Grande do Sul
90001, Portc Alegre, RS

The problem of initialization with the intent of eliminating spurious high-frequency gravitational oscillations by appropriate selection of the initial value in atmospheric models is a topic of considerable current interest. The available nonlinear initialization techniques [1], [4], [6] amount to decompose the model equations into a slow subsystem (Rossby subspace) and a fast one (gravity subspace) or to derive relationships under the premise that for a slow motion a certain number of its derivative must be also slow. See also [3].

An alternative method has been recently proposed by Lynch [5] through the use of the Laplace transform. The advantage being that its application does not require explicit knowlegde of the normal modes of the model. This is particularly important for cases on which the Coriolis parameter is considered in such a way that the horizontal spatial variables are nonseparable. Many limited-area barotropic models will give rise to such situation. Lynch's method is described below in a compact form. The formal solution of the weakly nonlinear evolution equation

$$u_t + L(u) + \epsilon N(u) = 0, \; u(0) = u_o, \qquad (1)$$

where E is a small parameter (Rosshy) is written as

$$u(t) = \frac{1}{2\pi i} \int_{\Gamma} e^{st} \, (sI+L)^{-1} \, (u_o - \epsilon \hat{N}(u))ds$$

Here Γ is a line parallel to the imaginary axis and $\hat{N}$ the transform of the nonlinearity. The filtered initial value is obtained through the iterative process

$$u^{(k+1)}(o) = \frac{1}{2\pi i} \int_{\Gamma^*} e^{st} \, (sI+L_N)^{-1} \, (u_o{}^k - E \, N(u_o{}^k)/s)ds \Big|_{t=0} \qquad (2)$$

on which L_N denotes a matrix discretization of the spatial linear operator L, Γ^* a closed contour that excludes the poles that give rise to higher frequencies, and $N(u_o{}^k)$ is an approximation to $\hat{N}(u)$ for t close to zero. The first approximant is chosen with $\epsilon = 0$. The integral is then approximated by quadrature over a circunscrihed polygon to the contour and letting e^{st} to be a Taylor polynomial.

We consider the ALG 619 numerical Laplace transform inversion method [8] instead of the finite contour approach used hy Lynch. It is particularly suitable for equatorial models on which there is a lack of strong frequency separation of the spectrum. The cutoff frequency would not exclude the presence of Kelvin or inertial-gravity waves. The spatial discretization is made with finite-differences and spectral methods. The use of semi-lagrangian techniques is under study. By using arguments similar to those in [2] we justify the Taylor approximation of e^{st} used hy Lynch on the quadrative of (2).

The first author acknowlegdes fruitful discussions with Pedro Dias Silva Leite from USP-CEPETEC, Claudio Pereira and Marco A. Lemes from CEPETEC and the support of its Director Dr. Gylvan Miera Filho.

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