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Basic procedures applicable to the real time solution of a class of multivariable functions with parallel processing resources

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BASIC PROCEDURES APPLICABLE TO THE REAL TIME SOLUTION OF A CLASS OF MULTIVARIABLE FUNCTIONS WITH PARALLEL PROCESSING RESOURCES

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SUMMARY

The basis for a methodology which leads to procedures which are adequate for efficient, real time solution of a class of multivariable functions in a parallel computer environment is introduced in this work, with emphasis on aspects which support adequate partition of a problem and the corresponding parallel processing resource alocation.

1. INTRODUCTION

This work presents considerations which lead to the formulation of a methodology applicable to the real time solution of a class of multivariable functions, relying on computer resources with parallel processing capabilities. The methodology which is presented considers the Finite Difference Method (FDM) serving as the basis for the mathematical solution of the CLASS OF PARABOLIC FUNCTIONS. This work also introduces considerations on criteria for partition of multivariable problems which would belong to the mentioned class of functions, such that an optimizing allocation of parallel computer resources can be achieved when they have to be solved. The mentioned criteria are used as the basis for presenting a formulation of structured procedures which are to be effectively employed in the solution of a problem.

2. THE ARCHITECTURE OF THE PARALLEL COMPUTER

The basic parallel computer architecture considered in the development of the procedures which are introduced in this work, is presented in Figure 1 [1,2].

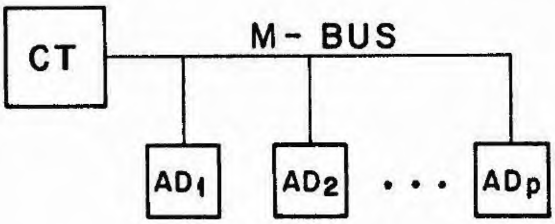


Fig. 1 - The parallel computer architecture.

This parallel computer architecture is based on three basic entities:

(1) The Controller (CT): The master unit with an arbiter, which manages the use of the common bus (M-BUS) by the parallel processors, while being the main I/O with the peripheral equipments;

(2) The Digital Analyzer (AD): The unit which executes the partition of the problem under its allocation;

(3) The M-BUS: The unit which connects all the parallel processing units, including the Controller.

The communication among the processing units is based on the principle of BROADCAST. The principle of data broadcast through the M-BUS is very convenient in iterative processes, moreover, where real time is a constraint. In this case, a data real-out from a processing unit can be, simultaneously, received by all the other processing units, which in turn, can use this information for updating, for instance, state variables which may be associated to the iterative process [1].

3. CRITERIA FOR A METHODOLOGY

A basic objective of the optimizing methodology in the partitioning of a problem to be executed by the parallel computer which was introduced in the preceding section (Figure 1) is the characterization of a set of sub-tasks, to be represented by specific Procedures, which can result in their equitable allocation among the parallel processing resources. Therefore, the method, as whole, characterizes a unique task which is partitionable.

Another basic concept of the methodology under consideration is concerned with characterization of two basic resources of a computer, in general: 1) Processing and; 2) Storage. More specifically, it is also implied that storage may have, in its turn, two basic attributes: 1) Storage for local use by a processor; or 2) Storage for communication with other processor(s).

Two optimizing criteria are considered of fundamental importance in the methodology:

(1) OPTIMIZATION OF THE BALANCE OF DATA PROCESSING AND STORAGE among the parallel processor of the computer by proper dimensioning of the sub-tasks which solve a problem;

(2) MINIMIZATION OF DATA PROCESSING FOR COMMUNICATION among the parallel processors of the computer.

Bearing in mind the two criteria which have just been mentioned, the next section of this work introduces the basic concepts which justify applicable PROCEDURES which can be derived from the methodology under development.

4. BASIC TOOLS FOR THE SPECIFICATION OF PROCEDURES

The basic concepts which specify the procedures are oriented for the solution of a class of problems in a domain Ω with a smooth boundary $\partial\Omega$.

The Finite Difference Method (FDM). It relies on the following concepts [3]:

- Discrete sampling of a continuous domain (Ω) in a finite number of mesh points (Figure 2) which, in turn, are denominated NODAL POINTS (NPi's);

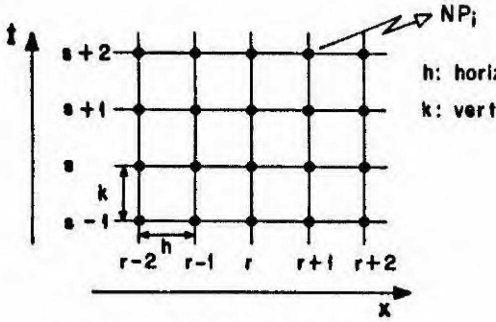


Fig. 2 - A mesh.

- Approximation of the solution of the problem, at a generic nodal point NPi, by a finite difference approximation. In the case of the current work the approximation obtained from an expansion of the Taylor series is being used for the class of function under consideration;
- Use of STENCILS which are characterized by specific configurations of a generic point NPi with a certain number of its neighboring nodal points in the mesh, depending on the approximation method being used, to solve the problem (Figure 3).

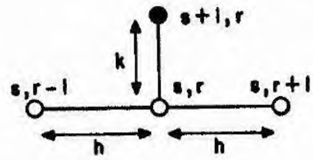


Fig. 3 - A stencil.

Decomposition of a Discrete Domain ΩD in Sub-Domains. The scheme represented in Figure 4 introduces basic entities which are defined from the sampling of a Domain Ω in a mesh, resulting in a Discrete Domain ΩD.

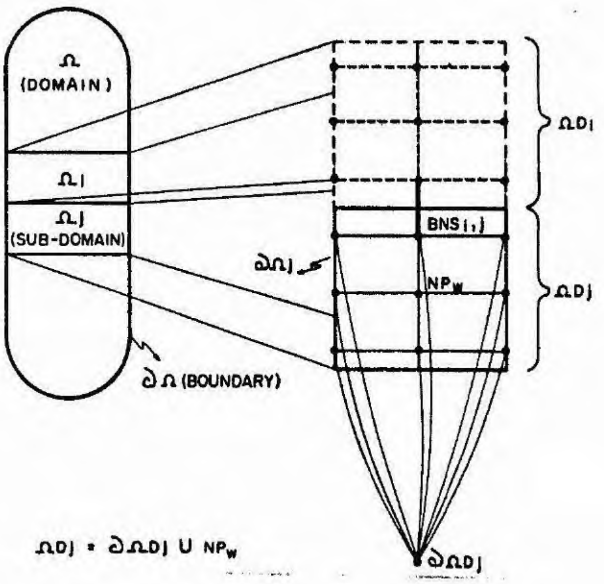


Fig.4 - Decomposition of Sub-Domains derived from ΩD.

Ω - Continuous Domain;
∂Ω - Boundary of a continuous domain;
ΩD - Discrete (sampled) Domain of Ω;
∂ΩD - Discrete Boundary of ∂Ω;
ΩDj - Generic Discrete Sub-domain, obtained from the partitioning of ΩD;
∂ΩDj - Discrete Boundary of sub-domain ΩDj;
BNSi,j - Boundary Nodal Segment connecting the nodal points NPi and NPj belonging, respectively, to the contiguous ΩDi and ΩDj sub-domains.

When the discrete points of the mesh do NOT lie at the nodal points (crossing of the mesh) of the discrete boundary of a sub-domain (∂ΩDj), they are defined as IRREGULAR [3] (discrete) nodal points, and they result from the crossing of a continuous boundary (∂Ω) with the mesh nodal segments at points other than the so called regular nodal points of the mesh. This principle is illustrated in Figure 5 and it is utilized as the basic diagram for stencils which can calculated the solution of the problem at these irregular nodal points.

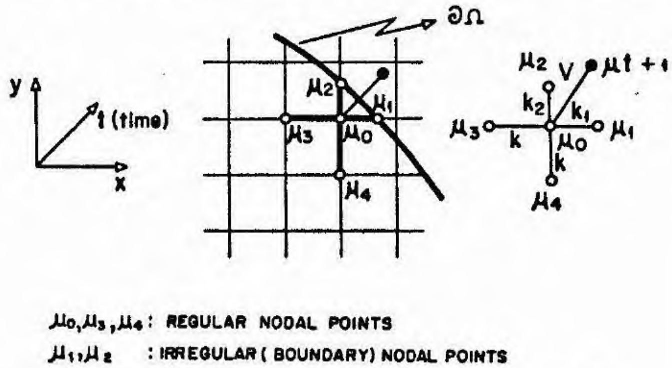


Fig.5 - Rectangular spacing with irregular nodal points in the boundary.

Solution with Finite Difference Approximations. Two types of method can be used in the solution of a problem: 1) direct and; 2) iterative, depending on the stencil which is being used.

Optimizing Partition of a Discrete Domain. Assuming an homogeneous workload for the parallel processor in the solution of a problem all over the discrete domain ΩD, an optimizing partition [4] of it can be obtained provided that: (a) The Number of Nodal Points (N-NP) for each discrete sub-domain ΩD_i of ΩD is approximately the same; (b) The Number of Boundary Nodal Segments (N-BNS) between each pair of discrete sub-domains is minimized, therefore minimizing the need for communication among processors. It is observed that the minimization of the N-BNS in a ΩD_i sub-domain, depends on: (1) The FORMAT of the sub-domain and (2) The STENCIL which is being utilized. It is fundamental to note that N-BNS is a concrete measure of the communication dependability between different processors which solve the problem, respectively, in different, neighboring sub-domains. In this case the minimization of N-BNS is a direct benefit in the speed-up of the problem solution, with direct real time implications. The following example is a brief illustration of these optimizing aspects.

Example: If ΩD is given with a rectangular shaped format with "nxℓ" nodal points and a discrete boundary ∂ΩD, the decomposition of this domain in "m" pseudo-triangular formatted partitions as observed in Figure 6-(a) is considered, utilizing a 5-point stencil (as represented in Figure 6-(b)) and p processors, such that k = m/p, where **k** is an integer constant, whose value depends on the number of available processors.

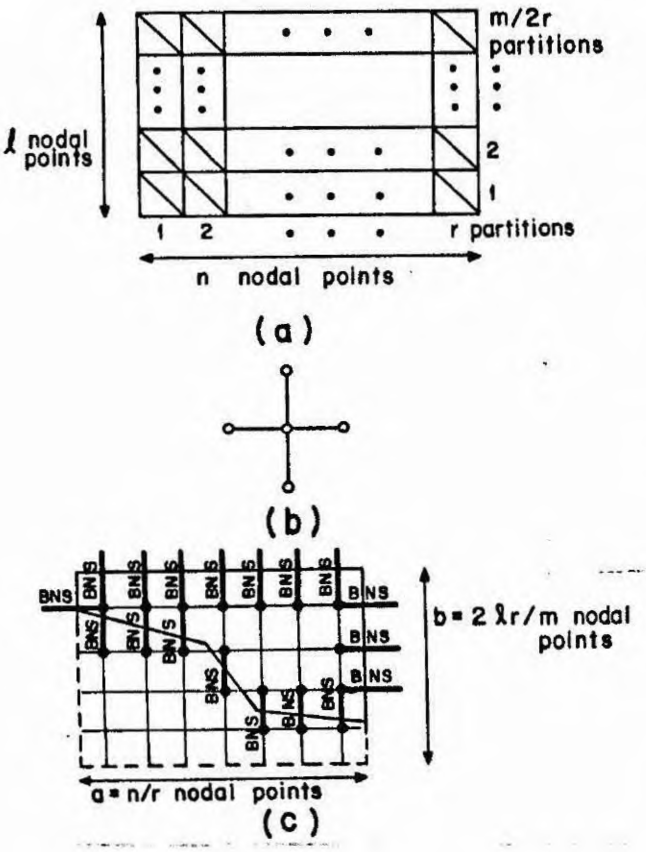


Fig. 6 – (a) Domain decomposition; (b) 5-point stencil; (c) identification of BNS's in a pseudo-triangular sub-domain.

The following conclusions can be drawn from Figure 6:

(1) The optimizing condition (a), as previously defined is satisfied;

(2) $N\text{-}BNS(r) = 2n/r + 2\ell r/m$;

(3) Minimizing the preceding expression, given in (2), in relation to r, the optimizing condition (b) will be satisfied, resulting:

$r = \sqrt{nm/\ell}$ and $N\text{-}BNS_{min} = 4\sqrt{n\ell/m}$;

(4) Considering the optimized value obtained for r, sides, a,b of the pseudo triangular sub-domain given in Figure 5-(c), result with values:

$a = \sqrt{n\ell/m}$ and $b = 2\sqrt{n\ell/m}$

The Optimizing Allocation of Sub-Tasks to the Processors. The basic principle which orient the optimizing allocation of Sub-Tasks to the processors of the parallel computer is the following: A Processor P should have allocated to it, a set of regular Sub-Domains $(\Omega D_i, \ldots, \Omega D_{i+k})$ and irregular Sub-domains $(\Omega D_j, \ldots, \Omega D_{j+q})$, in such a way that the Number of Boundary Nodal Segments(N-BNS), among these Sub-Domains, is the most extensive as possible. If this maximization can be obtained, the need for communication among the processor is minimized, as a result. This is so, because, for a fixed N-BNS, the more of a sub-set of them is allocated to a unique processor the less will be left in the boundary to be treated among the processors, provided that the same type of reasoning is equally applied for each processor of the set which form the parallel computer.

The basic tools for specification of a Procedure were given. In the following text, the concept of Procedure is formulated.

5. PROCEDURE (Pr)

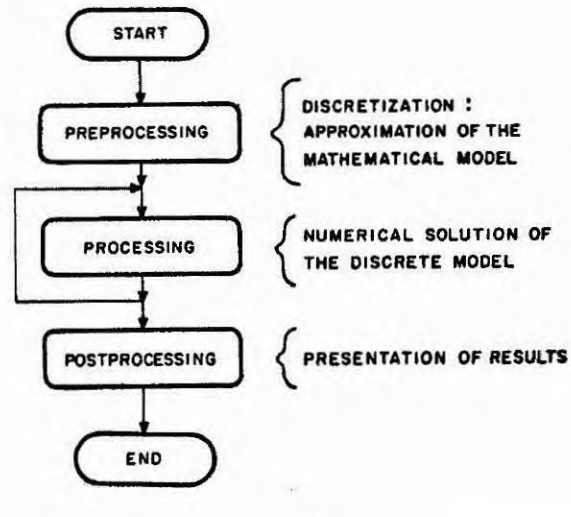
A Procedure (Pr) is defined by a set of structured actions, which are employed in the solution of a problem in a domain (Ω).

An interesting aspect related to the structuring of a procedure results in the characterization of "TEMPLATES", characterized as a pair of specific sub-domain formats and a specific stencils. A specific template would be drawn from a "table"containing a list of types of sub-domains and another "table" containing a repertoire with different types of stencils. Automatic procedures can be devised to select a convenient template, as defined above, to obtain the solution of a problem for each one of the sub-domains which cover the whole, complete domain where the problem is supposed to be solved.

There is a set of basic factors which are related to a Pr and, therefore, to the actions which it is involved with. These basic factors are: 1) PROBLEM-Which is supposed to be defined by a mathematical model (function to be solved), together with its initial and boundary conditions; 2) METHOD-Is defined by the type of numerical approximations which are employed, when the Finite Difference Method is utilized; 3) DOMAIN-Defined by its boundary points and by those points internal to the boundary; 4) SUB-DOMAINS-Defined by partitions of the Domain; 5) TECHNIC-Defined by the numerical algorithms which are used (ex. Jacobi, Gauss-Seidel, "SOR", etc).

The concepts introduced so far,are considered to be sufficient for the specification of an ordinated structure of actions which characterize a Procedure, this set of actions are represented in the fluxograms (a) and (b) of Figure 7, where (b) is a detailed representation of fluxograma (a).

It can be derived from the fluxograms of Figure 7 that a procedure is to be executed in three main steps.



(a)

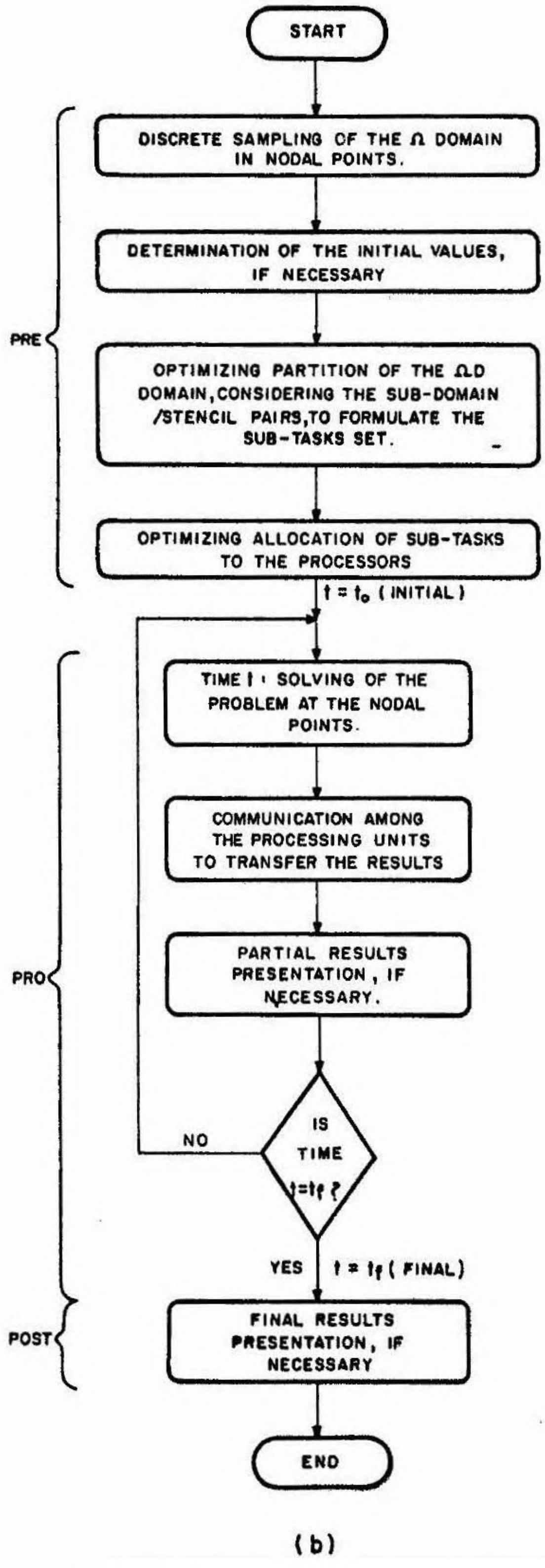


Fig. 7 - Top-down fluxogram of a Procedure.

6. CONCLUSION

This work presented the basic concepts and structures which are being used in the implementation of modular procedures to be employed in the solution of problems that can be characterized in a class of multivariable functions with the use of parallel computers.

It is expected that an appropriate application of the introduced procedure in a parallel processing environment result in the efficient use of the envolved computing resource. A partitioning criteria for task allocation in a parallel processing configuration is incorporated in the structure of the Procedure concept.

The introduced Methodology embodies concepts which make the resulting procedures adequate for real time processing. These concepts are being extended to other classes of problems, also oriented for real time parallel processing.

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